Empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA) represent a desperate attempt to break the suffocating hold on data analysis by the twin assumptions of linearity and stationarity. To analyze the data from nonlinear and non-stationary processes, various attempts such as Spectrograms, Wavelet analysis, and the Wigner-Ville distribution have been made, but the EMD-HSA approach is unique and different from the existing methods of data analysis. The EMD-HAS is truly an adaptive time-frequency analysis. It does not require an a priori functional basis. Instead, the basis functions are derived adaptively from the data by the EMD sifting procedures; the instantaneous frequencies are computed from derivatives of the phase functions of the Hilbert transform of the basis functions; the final result is presented in the time-frequency space. The EMD-HSA is a magnifying glass for analyzing the data from nonlinear and non-stationary processes. The EMD-HSA results are intriguing and are no longer shackled by spurious harmonics (the artifacts of imposing a linearity property on a nonlinear system) or limited by the uncertainty principle (the consequence of Fourier transform pairs in data analysis).

EMD-HSA was originally designed in 1995 specifically to study water surface wave evolution, the phenomenon of high frequency waves with short fetch evolving into low frequency waves at long fetch. With the EMD-HSA method, it was found that the evolution of the waves was not continuous but abrupt, discrete and local. Subsequently, NEH spent two years visiting Caltech at the invitation of Professor Theodore Y. Wu. Under the guidance of Professor Wu and Professor Owen M. Phillips of the Johns Hopkins University, the EMD-HSA method was further developed and various applications explored. Professor Wu designated the method as the Hilbert-Huang Transform (HHT), a name later adopted by NASA to avoid the awkward name of EMD-HSA. It is only fair to say that the HHT would not have been developed without the encouragement and guidance of Professors Wu and Phillips.

The HHT’s power and effectiveness in data analysis have been demonstrated by its successful application to many important problems covering engineering, biomedical, financial and geophysical data. The mathematical development of the HHT, however, is undergoing the same path as other significant and historical data analysis methods as in Fourier analysis and wavelet analysis: Applications are leading to development, and the mathematical theories are following, since the methods were motivated by applications. Mathematicians’ apparent interest in the HHT motivated our organization of an HHT mini-symposium at the joint meeting between

This book contains most of the presentations made at the mini-symposium with some additions. The book contents are divided into two groups: the theoretical aspects and the applications, with the applications further grouped into geophysics, structural safety, and visualization. In the theoretical aspects, the chapters cover the attempts of mathematicians to apply rigor to the empirical method such as the representation of the IMF by B-spline functions, filter based decompositions, and the statistical characteristics of the IMFs. This book also represents a plea for help from the mathematical community. A list of outstanding mathematical problems is given in Chapter 1. The chapters on applications include the correction of satellite orbit drifting, detection of failure of highway bridges and other structures, discoveries of the patterns and anomalies in climate data, and calculation of the instantaneous frequency of water waves. The objectives of the book are to provide HHT users with a collection of successful HHT applications, to supply graduate students and researchers with an HHT tutorial, and to inform data analysis mathematicians of the outstanding mathematical problems of HHT.

This book is intended as a reference for anyone who are involved in signal analysis by processing data from nonlinear and non-stationary systems. Although each chapter is independent from the others, it is sufficiently pedagogical so that every single chapter or the entire book is suitable as a part of a graduate course on signal analysis. To use this book efficiently, the readers should have background knowledge of calculus, Fourier transform, numerical analysis and differential equations. The HHT algorithm has been patented by NASA; non-commercial users may obtain it at the website: http://techtransfer.gsfc.nasa.gov.

Much effort went into compiling this collection of papers into a book form. In this processes, we owe our gratitude to Dr. Dean Duffy for his skillful editing and typesetting, and without his efficient and professional work, this book would not have been possible.

Norden E. Huang and Samuel S. P. Shen
Greenbelt, Maryland
CONTENTS

Preface v

Theoretical Aspects

1 Introduction to the Hilbert–Huang Transform and Its Related Mathematical Problems
Norden E. Huang

1.1 Introduction ........................................... 1
1.2 The Hilbert–Huang transform .......................... 2
   1.2.1 The empirical mode decomposition method (the sifting process) 4
   1.2.2 The Hilbert spectral analysis ..................... 12
1.3 Recent developments .................................. 14
   1.3.1 Normalized Hilbert transform ................. 15
   1.3.2 Confidence limit ............................ 17
   1.3.3 Statistical significance of IMFs ............ 18
1.4 Mathematical problems related to the HHT .......... 18
   1.4.1 Adaptive data-analysis methodology .......... 19
   1.4.2 Nonlinear system identification ............... 19
   1.4.3 The prediction problem for nonstationary processes (the end effects of EMD) .................. 20
   1.4.4 Spline problems (the best spline implementation for HHT, convergence and 2-D) .......... 21
   1.4.5 The optimization problem (the best IMF selection and uniqueness mode mixing) .......... 22
   1.4.6 Approximation problems (the Hilbert transform and quadrature) 23
   1.4.7 Miscellaneous statistical questions concerning HHT 24
1.5 Conclusion ........................................... 24

2 B-Spline Based Empirical Mode Decomposition
Sherman Riemenschneider, Bao Liu, Yuesheng Xu and Norden E. Huang

2.1 Introduction ........................................... 27
2.2 A B-spline algorithm for empirical mode decomposition .......... 29
2.3 Some related mathematical results .................... 33
## Contents

2.4 Performance analysis of BS-EMD .............................................. 39
2.5 Application examples ......................................................... 45
2.6 Conclusion and future research topics ...................................... 51

3 EMD Equivalent Filter Banks, from Interpretation to Applications 57
   *Patrick Flandrin, Paulo Gonçalves and Gabriel Rilling*

3.1 Introduction ................................................................. 57
3.2 A stochastic perspective in the frequency domain ...................... 58
   3.2.1 Model and simulations ............................................... 58
   3.2.2 Equivalent transfer functions .................................... 59
3.3 A deterministic perspective in the time domain ......................... 63
   3.3.1 Model and simulations ............................................... 63
   3.3.2 Equivalent impulse responses .................................... 63
3.4 Selected applications ...................................................... 64
   3.4.1 EMD-based estimation of scaling exponents ....................... 64
   3.4.2 EMD as a data-driven spectrum analyzer ......................... 68
   3.4.3 Denoising and detrending with EMD ............................... 69
3.5 Concluding remarks ....................................................... 73

4 HHT Sifting and Filtering ................................................... 75
   *Reginald N. Meeson*

4.1 Introduction ................................................................. 75
4.2 Objectives of HHT sifting ................................................ 77
   4.2.1 Restrictions on amplitude and phase functions .................. 78
   4.2.2 End-point analysis .................................................. 81
4.3 Huang's sifting algorithm ................................................. 81
4.4 Incremental, real-time HHT sifting ...................................... 82
   4.4.1 Testing for iteration convergence ................................ 83
   4.4.2 Time-warp analysis ................................................ 84
   4.4.3 Calculating warped filter characteristics ....................... 85
   4.4.4 Separating amplitude and phase .................................. 86
4.5 Filtering in standard time ................................................. 87
4.6 Case studies ............................................................... 89
   4.6.1 Simple reference example ......................................... 89
   4.6.2 Amplitude modulated example .................................... 90
   4.6.3 Frequency modulated example .................................... 92
   4.6.4 Amplitude step example .......................................... 95
   4.6.5 Frequency shift example ........................................ 99
4.7 Summary and conclusions ................................................ 102
   4.7.1 Summary of case study findings .................................. 102
   4.7.2 Research directions ................................................. 103
## Contents

5 Statistical Significance Test of Intrinsic Mode Functions 107  
*Zhaohua Wu and Norden E. Huang*

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>Introduction</td>
<td>107</td>
</tr>
<tr>
<td>5.2</td>
<td>Characteristics of Gaussian white noise in EMD</td>
<td>109</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Numerical experiment</td>
<td>110</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Mean periods of IMFs</td>
<td>110</td>
</tr>
<tr>
<td>5.2.3</td>
<td>The Fourier spectra of IMFs</td>
<td>111</td>
</tr>
<tr>
<td>5.2.4</td>
<td>Probability distributions of IMFs and their energy</td>
<td>113</td>
</tr>
<tr>
<td>5.3</td>
<td>Spread functions of mean energy density</td>
<td>116</td>
</tr>
<tr>
<td>5.4</td>
<td>Examples of a statistical significance test of noisy data</td>
<td>119</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Testing of the IMFs of the NAOI</td>
<td>120</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Testing of the IMFs of the SOI</td>
<td>122</td>
</tr>
<tr>
<td>5.4.3</td>
<td>Testing of the IMFs of the GASTA</td>
<td>123</td>
</tr>
<tr>
<td>5.4.4</td>
<td><em>A posteriori</em> test</td>
<td>125</td>
</tr>
<tr>
<td>5.5</td>
<td>Summary and discussion</td>
<td>125</td>
</tr>
</tbody>
</table>

5.3.1 Application to Geophysics

6 The Application of Hilbert–Huang Transforms to Meteorological Datasets 129  
*Dean G. Duffy*

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Introduction</td>
<td>129</td>
</tr>
<tr>
<td>6.2</td>
<td>Procedure</td>
<td>131</td>
</tr>
<tr>
<td>6.3</td>
<td>Applications</td>
<td>136</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Sea level heights</td>
<td>136</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Solar radiation</td>
<td>139</td>
</tr>
<tr>
<td>6.3.3</td>
<td>Barographic observations</td>
<td>142</td>
</tr>
<tr>
<td>6.4</td>
<td>Conclusion</td>
<td>145</td>
</tr>
</tbody>
</table>

7 Empirical Mode Decomposition and Climate Variability 149  
*Katie Coughlin and Ka Kit Tung*

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Introduction</td>
<td>149</td>
</tr>
<tr>
<td>7.2</td>
<td>Data</td>
<td>150</td>
</tr>
<tr>
<td>7.3</td>
<td>Methodology</td>
<td>152</td>
</tr>
<tr>
<td>7.4</td>
<td>Statistical tests of confidence</td>
<td>154</td>
</tr>
<tr>
<td>7.5</td>
<td>Results and physical interpretations</td>
<td>157</td>
</tr>
<tr>
<td>7.5.1</td>
<td>Annual cycle</td>
<td>158</td>
</tr>
<tr>
<td>7.5.2</td>
<td>Quasi-Biennial Oscillation (QBO)</td>
<td>159</td>
</tr>
<tr>
<td>7.5.3</td>
<td>ENSO-like mode</td>
<td>159</td>
</tr>
<tr>
<td>7.5.4</td>
<td>Solar cycle signal in the stratosphere</td>
<td>160</td>
</tr>
<tr>
<td>7.5.5</td>
<td>Fifth mode</td>
<td>161</td>
</tr>
</tbody>
</table>
Contents

7.5.6 Trends .................................. 162
7.6 Conclusions .................................. 162

8 EMD Correction of Orbital Drift Artifacts in Satellite Data Stream 167
Jorge E. Pinzón, Molly E. Brown and Compton J. Tucker

8.1 Introduction .................................. 167
8.2 Processing of NDVI imagery ......................... 169
8.3 Empirical mode decomposition ....................... 172
8.4 Impact of orbital drift on NDVI and EMD-SZA filtering ........... 173
8.5 Results and discussion ............................ 176
8.6 Extension to 8-km data ............................ 180
8.7 Integration of NOAA-16 data ....................... 181
8.8 Conclusions .................................. 183

9 HHT Analysis of the Nonlinear and Non-Stationary Annual Cycle of Daily Surface Air Temperature Data 187
Samuel S. P. Shen, Tingting Shu, Norden E. Huang, Zhaohua Wu,
Gerald R. North, Thomas R. Karl and David R. Easterling

9.1 Introduction .................................. 187
9.2 Analysis method and computational algorithms ................. 191
9.3 Data ........................................ 194
9.4 Time analysis .................................. 195
9.4.1 Examples of the TAC and the NAC ................... 195
9.4.2 Temporal resolution of data ....................... 197
9.4.3 Robustness of the EMD method ..................... 200
9.4.3.1 EMD separation of a known signal in a synthetic data set 200
9.4.3.2 Robustness with respect to data length .......... 200
9.4.3.3 Robustness with respect to end conditions ...... 202
9.5 Frequency analysis ................................ 202
9.5.1 Hilbert spectra of NAC ......................... 202
9.5.2 Variances of anomalies with respect to the NAC and TAC .... 204
9.5.3 Spectral power of the anomalies with respect to the NAC and TAC .......................... 205
9.6 Conclusions and discussion ........................ 207

10 Hilbert Spectra of Nonlinear Ocean Waves 211
Paul A. Hwang, Norden E. Huang, David W. Wang, and
Jame M. Kaihatu

10.1 Introduction .................................. 211
10.2 The Hilbert–Huang spectral analysis .................... 212
Contents

10.3 Spectrum of wind-generated waves ........................................... 216
10.4 Statistical properties and group structure ................................. 219
10.5 Summary .................................................................................. 222

Applications to Structural Safety

11 EMD and Instantaneous Phase Detection of Structural Damage .................................................. 227
Liming W. Salvino, Darryll J. Pine, Michael Todd and Jonathan M. Nichols

11.1 Introduction to structural health monitoring .................................. 227
11.2 Instantaneous phase and EMD ....................................................... 230
11.2.1 Instantaneous phase .............................................................. 230
11.2.2 EMD and HHT ................................................................. 231
11.2.3 Extracting an instantaneous phase from measured data ......... 233
11.3 Damage detection application ..................................................... 234
11.3.1 One-dimensional structures .................................................... 236
11.3.2 Experimental validations ....................................................... 239
11.3.3 Instantaneous phase detection ................................................. 242
11.4 Frame structure with multiple damage ......................................... 243
11.4.1 Frame experiment ................................................................. 244
11.4.2 Detecting damage by using the HHT spectrum .................... 247
11.4.3 Detecting damage by using instantaneous phase features .... 249
11.4.4 Auto-regressive modeling and prediction error .................. 252
11.4.5 Chaotic-attractor-based prediction error ......................... 255
11.5 Summary and conclusions ......................................................... 258

12 HHT-Based Bridge Structural Health-Monitoring Method .......................................................... 263
Norden E. Huang, Kang Huang and Wei-Ling Chiang

12.1 Introduction .............................................................................. 263
12.2 A review of the present state-of-the-art methods .................... 265
12.2.1 Data-processing methods .................................................... 266
12.2.2 Loading conditions ............................................................. 268
12.2.3 The transient load .............................................................. 270
12.3 The Hilbert–Huang transform ..................................................... 271
12.4 Damage-detection criteria ......................................................... 272
12.5 Case study of damage detection ................................................. 274
12.6 Conclusions ............................................................................. 280
Contents

Applications to Visualization

13 Applications of HHT in Image Analysis .......................... 289

Steven R. Long

13.1 Introduction .................................................. 289
13.2 Overview ...................................................... 290
13.3 The analysis of digital slope images ............................ 291
   13.3.1 The NASA laboratory .................................... 291
   13.3.2 The digital camera and set-up ............................ 292
   13.3.3 Acquiring experimental images ............................ 293
   13.3.4 Using EMD/HHT analysis on images ....................... 293
   13.3.5 The digital camera and set-up ............................ 293
      13.3.5.1 Volume computations and isosurface visualization ... 296
      13.3.5.2 Use of EMD/HHT in image decomposition ......... 300
13.4 Summary ...................................................... 303

Index .............................................................. 307