NONSTATIONARY SIGNALS ANALYSIS BY TEAGER-HUANG TRANSFORM (THT)

Jean-Christophe Cexus and Abdel-Ouahab Boudraa

IRENav, Ecole Navale/E^{3}I^2, ENSIETA
Ecole Navale, Lanvéoc-Poulmic, BP 600, Brest-Armées 29240, France.

ABSTRACT

In this paper a new method called Teager-Huang Transform (THT) for analysis of nonstationary signals is introduced. The THT estimates the Instantaneous frequency (IF) and the Instantaneous amplitude (IA) of a signal. The method is based on the Empirical mode decomposition (EMD) algorithm of Huang *et al.* [1] and the Teager energy operator (TEO) [2]. Both EMD and TEO deal with non-stationary signals [1],[3]. The signal is first band pass filtered using the EMD into zero-mean AM-FM components called Intrinsic mode functions (IMFs) with well defined IF. Then TEO tracks the modulation energy of each IMF and estimates the corresponding IF and IA. The final presentation of the IF and the IA results is an energy Time frequency representation (TFR) of the signal. Based on the EMD, the THT is flexible enough to analyze any data (linear or nonlinear) and stationary or nonstationary signals. Furthermore, the THT is free of interferences. To show the effectiveness of the THT, TFRs of five synthetic signals are presented and results compared to those of the Hilbert-Huang transform (HHT) [1], the spectrogram, the Smoothed pseudo Wigner-Ville distribution (SPWVD), the scalogram and the Choi-Williams distribution (CWD).

1. INTRODUCTION

Time-frequency (TF) analysis is an effective tool for analyzing nonstationary signals, i.e., signals whose spectral contents vary with time [4]. In many areas such as in seismic, radar, sonar, telecommunications and biomedicine signals under consideration are known to be nonstationary. An important feature of nonstationary signal is provided by its IF, which accounts for the spectral variations as a function of time [4]. A good way to define the local characteristics of nonstationary signal is its IF. In TF analysis, we analyze the frequency content a cross a small span of time and then move to another time position [5]. The major drawback of most TF transforms is that the rectangular tiling of TF plane does not match the shape of all signals [1]. On the other hand, basis decomposition methods such as the Fourier decomposition or the wavelet decomposition have been used to analyze real signals. The main drawback of these approaches is that the basis functions are fixed, and do not necessarily match varying nature of signals. The IF of an analytic signal is commonly defined as the first derivative of the phase of this signal. Quantitatively, the IF is a time varying parameter which corresponds to the frequency of a sinusoid that locally (in time) fits the signal under analysis. The IF if blindly applied to any analytic signal, may result in few paradoxes such as [7]: (1) the IF may not be one of this frequencies in the Fourier spectrum, (2) the IF may go outside the band for a band limited signal. The signal that is symmetric with respect to the local zero mean and has the same number of zero crossings and extrema gives a meaningful IF [1]. Recently, Huang *et al.* [1] have proposed a class a functions (oscillating components) called Intrinsic mode functions (IMFs). To compute the frequency behavior of each IMF in time, the IF is estimated. A standard approach to this problem is to use the Hilbert transform (HT) and the related Gabors analytic signal [4]. An alternative approach developed by Maragos *et al.* [2] uses an energy-tracking operator, Teager energy operator (TEO), to first estimate the energy required for generating an AM-FM signal and then separate it into its IF and IA components. Note that the HT approach mainly involves a linear integral operator, where as the TEO approach uses a non-linear differential operator. TEO gives a good estimate of IF and has low computational complexity. In the presented work, the EMD is used in conjunction with the TEO to estimate the IF and the IA of a given signal and to generate a full enegry-frequency-time distribution of this signal. This distribution is free of interferences.

2. EMD ALGORITHM

The principle of the EMD is to decompose adaptively a given signal into IMFs extracted from the signal by means of the sifting algorithm. The name IMF is adapted because it represents the oscillation mode embedded in the data. With this definition, the IMF in each cycle, defined by the zero crossings of, involves only one mode of oscillation, no complex riding waves are allowed. Thus, an IMF is not restricted to a narrow band signal, and it can be both amplitude and frequency modulated. In fact, it can be nonstation-

e-mail:(boudra, cexus)@ecole-navale.fr

ary. The essence of the EMD is to identify the IMF by characteristic time scales, which can be defined locally by the time lapse between two extrema of an oscillatory mode or by the time lapse between two zero crossings of such mode. The EMD picks out the highest frequency oscillation that remains in the signal. Thus, locally, each IMF contains lower frequency oscillations than the one extracted just before. Furthermore, the EMD does not use any pre-determined filter or wavelet function. It is fully data driven method. It has been shown experimentally that the EMD acts essentially as dyadic filter bank resembling those involved in wavelet decomposition [6]. Since the decomposition of the EMD is based on the local characteristics time scale of the data, it is applicable to nonlinear and nonstationary processes. The EMD decomposes into a sum of IMFs that : (R1) have the same numbers of zero crossings and extrema; and (R2) are symmetric with respect to the local mean. The First condition is similar to the narrow-band requirement for a stationary Gaussian process. The second condition modifies a global requirement to a local one, and is necessary to ensure that the IF will not have unwanted fluctuations as induced by a symmetric waveforms [1]. Both conditions satisfy the physically necessary conditions, IMFs, are obtained using the sifting algorithm involving the following steps :

Step 1) Fix ϵ, $j \leftarrow 1$ (j^{th} IMF)
Step 2) $r_{j-1}(t) \leftarrow x(t)$ (residual)
Step 3) Extract the (j^{th} IMF) :
 (a) $h_{j,i-1}(t) \leftarrow r_{j-1}(t), i \leftarrow 1$ (i number of sifts)
 (b) Extract local maxima/minima of $h_{j,i-1}(t)$
 (c) Compute upper envelope and lower envelope functions $U_{j,i-1}(t)$ and $L_{j,i-1}(t)$ by interpolating, using cubic spline, respectively local maxima and minima of $h_{j,i-1}(t)$
 (d) Compute the envelopes mean : $\mu_{j,i-1}(t) \leftarrow (U_{j,i-1}(t) + L_{j,i-1}(t))/2$
 (e) Update: $h_{j,i}(t) \leftarrow h_{j,i-1}(t) - \mu_{j,i-1}(t), i \leftarrow i+1$
 (f) Calculate stopping criterion :
$$SD(i) = \sum_{t=0}^{T} \frac{\mid h_{j,i-1}(t) - h_{j,i}(t) \mid^2}{(h_{j,i-1}(t))^2}$$
 (g) Decision : Repeat Step (b)-(f) until $SD(i) < \epsilon$ and then put $IMF_j(t) \leftarrow h_{j,i}(t)$ (j^{th} IMF)
Step 4) Update residual : $r_j(t) \leftarrow r_{j-1}(t) - IMF_j(t)$
Step 5) Repeat Step 3 with $j \leftarrow j+1$ until the number of extrema in $r_j(t) < 2$.

where T is the time duration of the signal. The sifting is repeated several times (i) in order to get h to be a true IMF that fulfills the requirements (R1) and (R2). The result of the sifting procedure is that $x(t)$ will be decomposed into $IMF_j(t), j = 1, \ldots N$ and residual $r_N(t)$, i.e., $x(t) = \sum_{j=1}^{N} IMF_j(t) + r_N(t)$. To guarantee that the IMF components retain enough physical sens of both amplitude and frequency modulations, a criterion for the sifting to stop is used. This accomplished by limiting the size of the standard deviation SD computed from the two consecutive sifting results. Usually, SD is set between 0.2 to 0.3 [1].

3. TEAGER ENERGY OPERATOR

It is shown that the TEO can track the energy and identify the IF and the IA of a signal [2]. The TEO, $\Psi[.]$, is defined for continuous-time signal $x(t)$ as :

$$\Psi[x(t)] = [\dot{x}(t)]^2 - x(t)\ddot{x}(t) \; ; \tag{1}$$

where $\dot{x}(t)$ and $\ddot{x}(t)$ are the first and the second time derivatives of $x(t)$ respectively. In the discrete case, the time derivatives may be approximated by time differences. The discrete-time counterpart of TEO becomes [2] :

$$\Psi(x[n]) = x^2[n] - x[n+1] \cdot x[n-1] \tag{2}$$

An important aspect of TEO is that it is nearly instantaneous. This is because only three samples are required for the energy computation at each time instant. This excellent time resolution provides us with ability to capture the energy fluctuations. Furthermore, this operator is very easy to implement efficiently. The Energy separation algorithm (ESA) developed par Maragos *et al.* [2] uses the TEO to separate $x(t)$ into its amplitude envelope $\mid a(t) \mid$ and IF signal $f(t)$ to accomplish monocomponent AM-FM signal demodulation :

$$f(t) \approx \frac{1}{2\pi}\sqrt{\frac{\Psi[\dot{x}(t)]}{\Psi[x(t)]}} \quad \mid a(t) \mid \approx \frac{\Psi[x(t)]}{\sqrt{\Psi[\dot{x}(t)]}} \; ; \tag{3}$$

Since the speech signal is composed of superposition of AM-FM signals, TEO has been successfully used in various speech processing applications [2]. The ESA is very simple demodulating technique for AM-FM demodulation. It is less computationally complex and has better time resolution than other classical demodulation approaches such as the HT [7]. The main disadvantage is of this operator is its sensitivity in very noise environment. Furthermore, it assumes that the estimated IF does not vary too fast (small bandwidths) or too greatly compared with the carrier frequency.

4. TEAGER-HUANG TRANSFORM: THT

The EMD is not a TFR as the WVD. With the HT, the IMF yields IFs as function of time that give sharp identifications of embedded structures. The final presentation of the results is an energy TFR, designated as Hilbert-Huang transform (HHT) [1]. In this work, to estimate the IF and the IA of $x(t)$, the EMD is combined with the TEO, which is typically applied to a bandpass signal. If $x(t)$ is a multicomponent AM-FM signal, then bandpass filtering is needed to isolate each component before applying the ESA. Thus, the EMD is used as a multiband filtering to separate the signal components in the temporal domain and hence reduce multicomponent demodulation to multicomponent one. The conjunction of the EMD and the TEO methods is designated as Teager-Huang Transform (THT). The final presentation of the the IF and the IA results is an energy TFR. The THT method, whose block diagram is shown in Figure 1, can be divided into two tasks; separation of the signal into IMFs using the EMD, and demodulation of the separated components (IMFs) into IF and IA information signals for each component using the ESA.

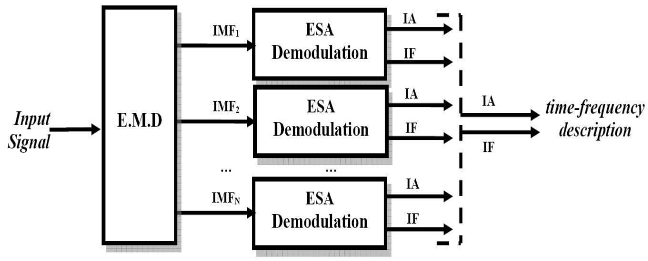


Fig. 1. Block diagram of the THT

5. RESULTS

The THT is illustrated by five test signals with IF laws shown in figure 2. The first signal, $s_1(t)$, consists of four lines in the TF domain (Fig. 2(a)): two LFM and two tones. The TFRs of the spectrogram, the SPWVD, the CWD, the scalogram, the HHT and the THT are shown in figure 3. Even if the spectrogram, the SPWVD, the CWD and the scalogram identify the four components (Figs. 3(a)-(d)), the signal component localization is coarse. The best result is given by the HHT and the THT (Figs. 3(e)-(f)): the four components are well identified and are much better localized, leading to nearly ideal TFR. Note that the second tone is more resolved in frequency by the THT than by the HHT. The second signal, $s_2(t)$, is composed of two hyperbolic FM signals (Fig. 1(b)). The TFRs of the SPWVD, the scalogram, the HHT and the THT are shown in figure 4. The four TFRs identify the two frequency components. In spite of the smoothing operation, the SPWVD shows cross-terms along the mean of the two hyperbola (Fig. 4(a)) and with loss of resolution well visible in time between $t = 850$ and $t = 900$ and in frequency between $\nu = 0.12$ and $\nu = 0.5$ for first hyperbola and between $\nu = 0.3$ and $\nu = 0.5$ for second hyperbola. The scalogram presents some interferences between the hyperbola and for $t > 600$ the two hyperbola are not separated. Comparing the THT and the HHT against the SPWVD and the scalogram, we see that in the THT and the HHT the two components are well localized with no cross-terms. For second hyperbola there is no loss of TF resolution. For first hyperbola there is loss of TF resolution between $\nu = 0.24$ and $\nu = 0.5$. The third signal, $s_3(t)$, is composed of a linear and hyperbolic FM signals, and two Gaussian atoms located at ($t = 425$, $\nu = 0.05$) and ($t = 800$, $\nu = 0.15$) respectively (Fig. 2(c)). The TFRs of the SPWVD, the scalogram, the HHT and the THT are shown in figure 5. The SPWVD and the scalogram identify the four IF laws but the signal components localization is coarse. Note that the spread in frequency is more important for the scalogram than for the SPWD. The HHT shows also spread in frquency for $\nu > 0.15$. For the THT the TF structures are clearly visible and highly concentrated with narrow spread in frequency. The fouth signal, $s_4(t)$, consists of five nonoverlapping tones of finite duration and a sinusoidal FM (Fig. 2(d)). The TFRs of the SPWVD, the scalogram, the HHT and the THT are shown in figure 6. With the THT both the tones and the sinusoidal FM are much better identified and localized (Fig. 6(d)) than the remaining TFRs. Note the spread in frequency, for the tones, in the HHT, the SPWD and the scalogram. For the SPWVD and the scalogram the transitions between tones are smooth (Figs. 6(a)-(b)). The fifth signal, $s_5(t)$, consists of a Dirac impulse, two tones and a LFM of finite durations (Fig. 2(e)). The TFRs of the SPWVD, the scalogram, the HHT and the THT are shown in figure 7. This figure shows that the THT gives the best result (Fig. 7(d)). The representations of the SPWVD and the scalogram are hardly readable with loss in frequency resolution and particularly for the SPWVD where cross-terms are superposed on the the signal components. For $t > 100$, we note a spread in frequency in the HHT.

6. CONCLUSION

In this paper a new full energy-frequency-time distribution of a signal called THT is proposed. This distribution free of interferences is designed for processing any data (linear or nonlinear) and stationary or nonstationary signals. The THT is illustrated by five synthetic signals with different IF laws and the results compared to those of well known TFRs. These results show that the THT identifies in all presented cases the TF structures. Furthermore, THT presents no cross-terms and few loss of TF resolution. The THT is very easy to implement. As future work we plan to study the THT in noisy environment and also the effects of the sampling and the interpolation, used by the EMD, on quality (resolution, concentration,...) of the generated TFRs.

7. REFERENCES

[1] N.E. Huang et al., ”The empirical mode decomposition and the Hilbert spectrum for nonlinear and nonstationary time series analysis,” *Proc. Royal Soc. London A*, vol. 454, pp. 903-995, 1998.

[2] P. Maragos, J.F. Kaiser and T. Quatieri, ”Energy separation in signal modulation to speech analysis,” *IEEE TSP.*, vol. 41, pp. 3024-3051, 1993.

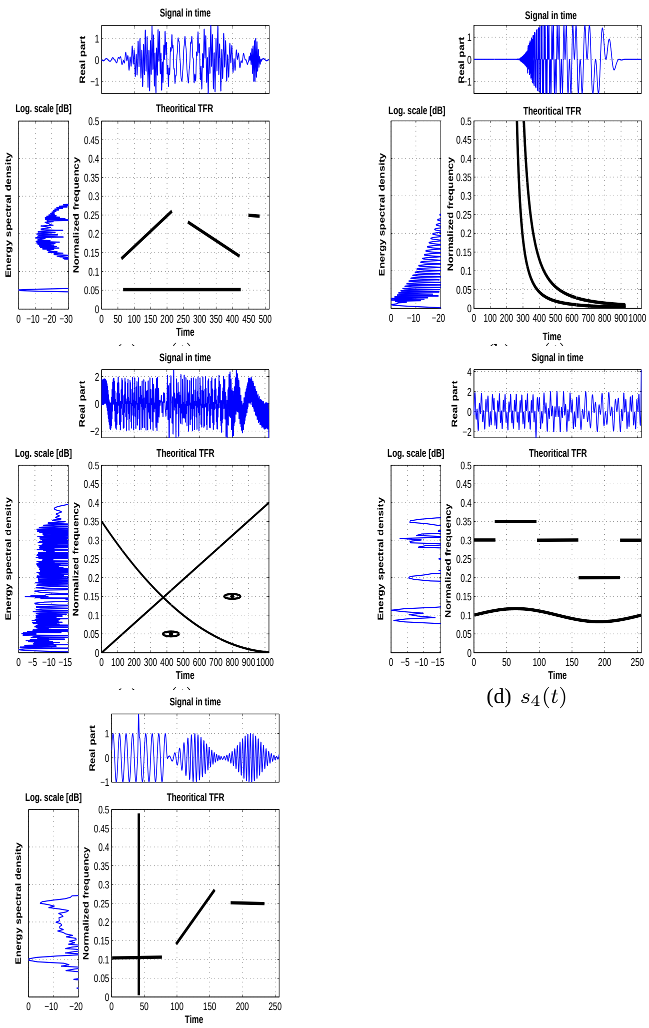
[3] J.C. Cexus and A.O. Boudraa, ”Link between cross-Wigner distribution and cross-Teager energy operator,” *IEE Elec. Lett.*, vol. 40, no. 12, pp. 778-780, 2004.

[4] B. Boashash, *Time-Frequency Signal Analysis and Processing* Elsevier, 2003.

[5] L. Cohen, *Time-Frequency Analysis*, Prentice-Hall, 1995.

[6] P. Flandrin, G. Rilling and P. Goncalves, ”EMD as a filter bank,” *IEEE SPL*, vol. 11, pp. 112-114, 2003.

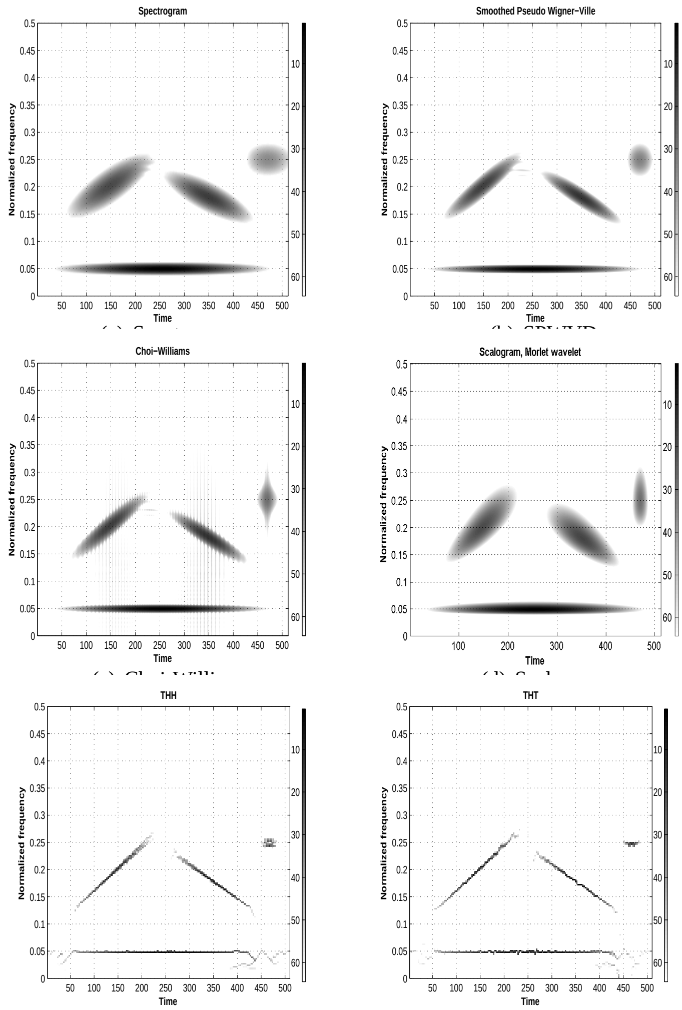
[7] A. Potamianos and P. Maragos, ”A comparison of the energy operator and Hilbert transform approach to signal ans speech demodulation,” *Sig. Process.*, vol. 37, pp. 95-120, 1994.



(d) $s_4(t)$

(e) $s_5(t)$

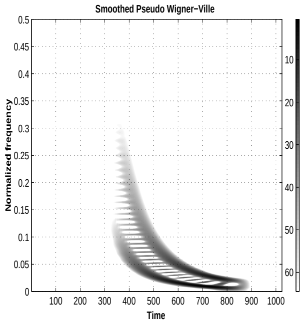
Fig. 2. Ideal TFRs of five test signals.



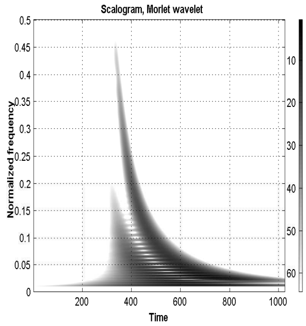
(e) HHT.

(f) THT.

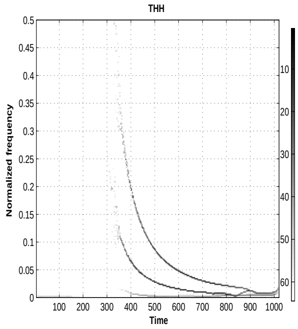
Fig. 3. Signal $s_1(t)$



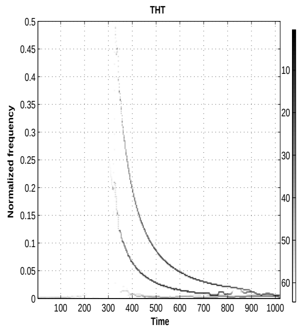
(a) SPWVD.



(b) Scalogram

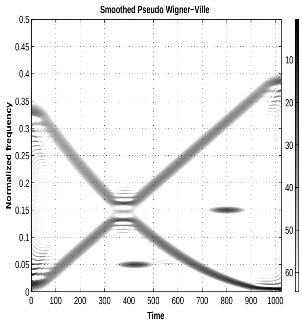


(c) HHT.

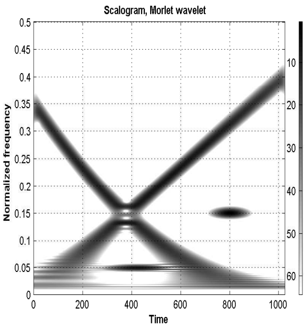


(d) THT.

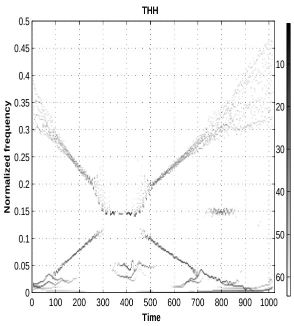
Fig. 4. Signal $s_2(t)$



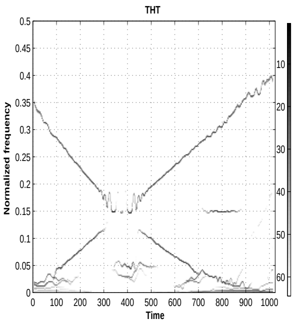
(a) SPWVD.



(b) Scalogram.

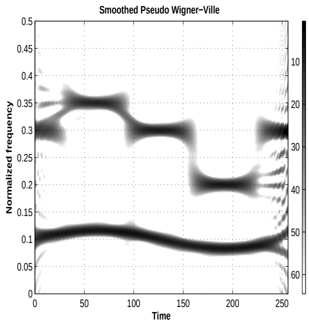


(c) HHT.

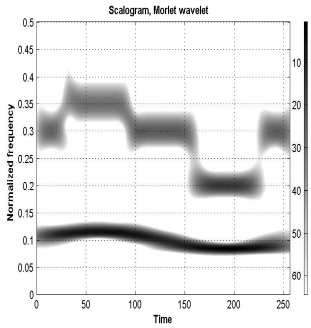


(d) THT.

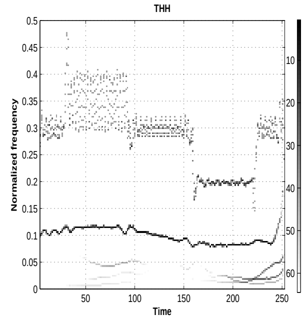
Fig. 5. Signal $s_3(t)$



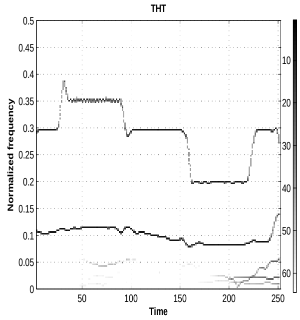
(a) SPWVD.



(b) Scalogram.

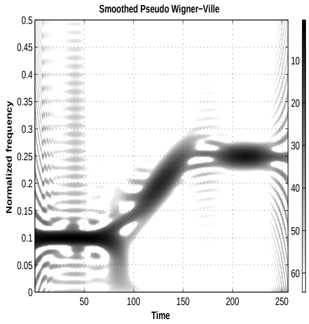


(c) HHT.

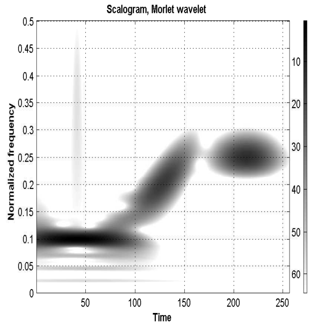


(d) THT.

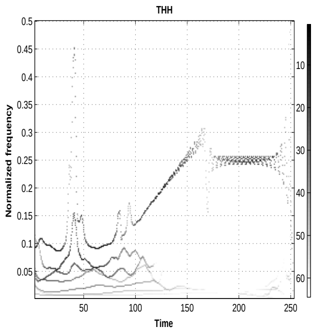
Fig. 6. Signal $s_4(t)$



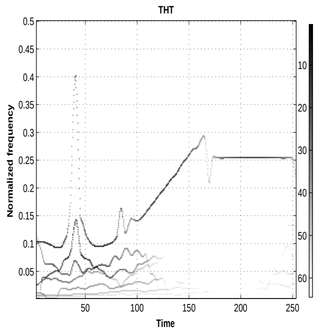
(a) SPWVD.



(b) Scalogram.



(c) HHT.



(d) THT.

Fig. 7. Signal $s_5(t)$