Denoising Electrical Signal via Empirical Mode Decomposition

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Abstract

Electric signals are affected by numerous factors, random events, and corrupted with noise, making them nonlinear and non-stationary in nature. In recent years, the application of Empirical Mode Decomposition (EMD) technique to analyze nonlinear and non-stationary signals has gained importance. It is an empirical approach to decompose a signal into a set of oscillatory modes known as intrinsic mode functions (IMFs). Based on an empirical energy model of IMFs, the statistically significant information content is established and combined. In this paper, we demonstrate an approach to detect power quality disturbances in noisy conditions. The approach is based on the statistical properties of fractional Gaussian noise (fGn).

1. Introduction

A signal obtained from any system is never a prefect reflection of the actual measurement. Rather, a signal is always corrupted by noise introduced by the device itself or by other means. Therefore, an observed signal is a combination of actual information and noise,

$$x(t) = s(t) + n(t) \qquad (1)$$

where $x(t)$ is the observed signal, $s(t)$ is the actual measurement, and $n(t)$ is the noise. The unknown amount and type of noise present in the data can lead to misinterpretation of the phenomena reflected by the signal.

In this paper, we study an electrical power signal corrupted with noise and disturbances. A 50 Hz sinusoidal voltage signal of 2 volts peak to peak amplitude is shown in Fig. 1. A constant supply of normal voltage with less than 10% variations is considered as good power supply quality. With increasing use of solid state switching devices, nonlinear load switching, rectifiers, inverters, and improper load balance, the power quality degrades. Thus, power quality monitoring has gained tremendous importance in recent years [1, 2]. Power disturbances based on effects, duration, and intensity can be divided into five categories, voltage fluctuations, transients, harmonics, power outages, and electrical noises. Among all the
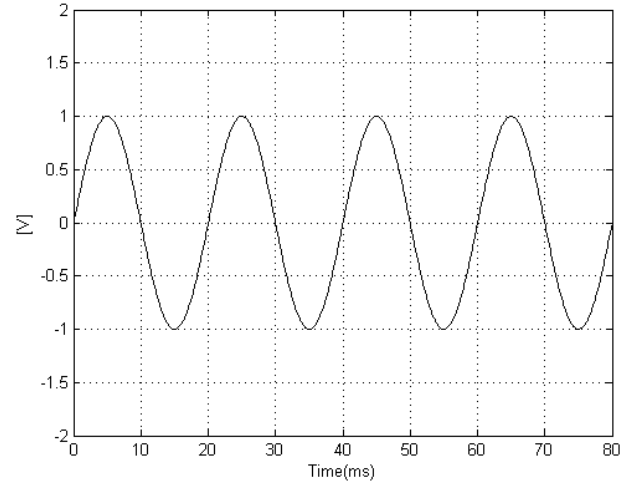

Fig. 1. Normal voltage signal.

disturbances, electrical noise is the most common.

Electrical noise is generally white noise. Consider a signal $z(t)$ of the form,

$$z(t) = x(t) + y(t) \qquad (2)$$

where $y(t)$ is the power disturbance. The detection of $y(t)$ in noisy conditions is a challenging task. The application of signal processing tools for noise removal and power quality monitoring is been studied extensively [3-5]. Wu et al. discusses the limitations of various existing denoising approaches in signal processing [6]. Wavelet transform based power quality monitoring received wide attention because of its ability to handle nonlinear and non-stationary signal [7-9]. Santoso et al. detected power disturbances using wavelets and classified then accordingly using pattern recognition techniques [10, 11].

In this paper, we demonstrate the performance of EMD in detecting power disturbance in noisy conditions. The EMD approach originally proposed by Huang et al. is a highly adaptive decomposition technique [12]. It decomposes any complicated signal into a finite set of functions called intrinsic

mode functions by the process known as *sifting*. The behavior of the EMD as a filter bank is highlighted by Wu et al. and Flandrin et al. in their analysis of noise [6, 13]. Lu et al studied EMD for power quality monitoring but failed to analyze the affect of noise and did not provide statistical evidence of their reconstruction [15]. We develop the fractional Gaussian noise energy model as described by Flandrin et al. for different Hurst (*H*) value [13, 14]. The fractional Gaussian noise is a generalization of white noise and $H = 0.5$ is a special case of fGn corresponding to white noise. Given the statistical energy model of noise and the signal $z(t)$, we detect the disturbance signal $y(t)$ by partial reconstruction of the IMFs.

This paper is organized as follows. Section 2 describes the fractional Gaussian noise. In Section 3, the EMD methodology is discussed. Experimental results showing denoising and detection of oscillatory transients in power signals are presented in Section 4. Finally, the conclusions and future work are presented in Section 5.

2. Fractional Gaussian Noise Model

Fractional Gaussian noise is a generalization of white noise [16]. It is closely linked with self-similar stochastic processes and random fractals both which have been extensively considered is signal processing applications. It is expressed as incremental process of fractional Brownian motion and its statistical properties are controlled by a single parameter, *H*, known as the Hurst exponent. The statistical properties of fGn ($X_{H,i}$) are mean zero, stationary Gaussian time series, and the auto-covariance function $\gamma_H(k) = E(X_{H,i} X_{H,i+k})$ is,

$$\gamma_H(k) = 2^{-1}\sigma^2 \{|k+1|^{2H} - 2k^{2H} + |k-1|^{2H}\}, k > 0 \quad (3)$$

where, σ^2 is the variance of fGn. The value of *H* is in the range 0 to 1. For a special case, $H = 0.5$ $X_{H,i}$ is white noise, while for $0 < H < 0.5$, $X_{H,i}$ is negatively correlated, and for $0.5 < H < 1$, $X_{H,i}$ is positively correlated. The Fourier transform of equation (3) gives the power spectral density of fGn,

$$S_H(f) = C_H \sigma^2 |e^{i2\pi f} - 1|^2 \sum_{k=-\infty}^{\infty} \frac{1}{|f+k|^{(2H+1)}} \quad (4)$$

Where, C_H is a constant and $|f| < 0.5$.

3. Empirical Mode Decomposition

Empirical mode decomposition proposed by Huang et al. [12] deals with nonlinear and non-stationary signals. It is an intuitive, direct, and adaptive approach as it derives the basis function from the signal itself unlike the Fourier transformation and Wavelets.

The intrinsic mode functions obtained from the decomposition of the signal $x(t)$ by EMD must obey two general assumptions; (i) each intrinsic mode must have the same number of extrema and zero crossings or differ at most by one and (ii) must be symmetric with respect to the local zero mean. These two assumptions assist in defining meaningful instantaneous frequency of an IMF. Based on these assumptions, the *sifting* procedure to obtain IMFs of the signal $x(t)$ is described as follows.

1) Identify all the maxima and the minima in the signal $x(t)$.
2) Generate its upper and lower envelopes using cubic spline interpolation.
3) Compute the point by point local mean m_1 from upper and lower envelopes.
4) Extract the details, $h_1 = x(t) - m_1$.
5) Check the properties of h_1 and iterate *k* times, then $h_{1k} = h_{1(k-1)} - m_{1k}$ becomes the IMF once it satisfies some stopping criterion. It is designated as first IMF $c_1 = h_{1k}$.
6) Repeat steps 1) to 5) on the extracted data $r_1 = x(t) - c_1$.
7) The step 6) is repeated until all the IMFs and residual is obtained.

The stopping criterion, the normalized squared difference between two successive *sifting* operations is defined as,

$$SD_k = \frac{\sum_{t=1}^{T} |h_{k-1}(t) - h_k(t)|^2}{\sum_{t=1}^{T} h_{k-1}^2} \quad (5)$$

The SD_k value is generally set between 0.2 and 0.3. The decomposed signal can be represented as,

$$x(t) = \sum_{n=1}^{N} c_j + r_n \quad (6)$$

where N is the total number of IMFs and r_n is the final residue which can be either the mean trend or a constant. EMD provides a general purpose time space filtering as,

$$x_{lq}(t) = \sum_{n=1}^{q} c_n(t) \quad (7)$$

where $l, q \in [1,...,N], l < q$. When $l = 1$ and $q < N$, it is a high-pass filtered signal, when $l > 1$ and $q = N$, it is a low-pass filtered signal, and when $1 < l \leq q < N$, it is a band-pass filtered signal.

4. Experimental Results

In our experimental analysis, we initially develop the fGn model for various H values as described by Flandrin et al. [14]. We generate $J = 1000$ samples of fGn of length $M = 2048$ for each H (0.1, 0.2, … , 0.9) values. EMD is computed for all the samples ($X_{H,i}^J, i = 1,.....,M$) for a given H values, resulting in a collection of IMFs ($c_{H,n}^J, n = 1,.....,N$). The maximum number of IMFs is not constant. However, each sample did not consist of less than 10 IMFs. Therefore, we use the first 10 IMFs to construct the noise model. The variance of the IMFs is,

$$V_H(n) := \text{var}(c_{H,n}(i)) \qquad (8)$$

The empirical variance (energy) estimate is given by,

$$\hat{V}_H(n) = \frac{1}{M} \sum_{i=1}^{M} c_{k,H}(i)^2 \qquad (9)$$

The noise model [14] is,

$$\hat{V}_H(n) = C_H \rho_H^{-2(1-H)n}, n \ge 2 \qquad (10)$$

where, $C_H = \hat{V}_H(1)/\beta_H$ and with $\rho_H \approx 2$. The value of β_H is $\rho_H^{(H-1)}$. The IMFs log variance is,

$$\log_2 \hat{V}_H(n) = \log_2 V_H(2) + 2(H-1)(n-2)\log_2 \rho_H, \\ n \ge 2 \qquad (11)$$

From equation (11), we observe that it is a simple linear model in semi-log plot of empirical variance against IMF indices. Given the model, we then eliminate the power disturbances based on a partial reconstruction of the IMFs.

4. 1 Electrical Signal Denoising

Electrical noise is high frequency interference caused by a number of factors, including arc welding or the operation of some electric motors. If left unnoticed, it may result in processing errors, burned circuit boards, degraded electrical insulations, and equipment damage. We corrupted a normal voltage signal with fGn ($H = 0.5$) generated using the Lowen approach [17] as shown in Fig. 2.

The approach adopted to eliminate electrical noise and obtain a normal signal is, in accordance with [14], as follows.
1) EMD decomposes the noisy signal into IMFs. Since the first IMF contains most of the high frequency, it is assumed to contain noise. The noise level of IMF c1 is obtained using equation (9).
2) The noise only model using equation (10) is obtained.

3) The confidence interval (95% and 99%) of the noise only model is obtained.
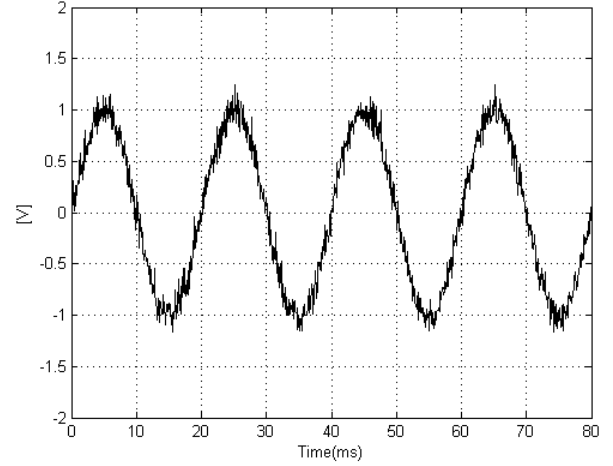4) The empirical energy model of the noisy signal is obtained using equation (11).

Fig. 2. Normal voltage signal corrupted with fractional Gaussian noise ($H = 0.5$).

5) The IMFs whose energy is above threshold, i.e., the confidence interval as said to contain the signal information and the signal is obtained by partial reconstruction of those IMFs and residual.

EMD of noisy signal provides IMFs c1 to c10 and residual component as shown in Fig. 3. Based on the procedure described above, the noisy signal energy model and the noise only model, along with 95% and 99% confidence interval are shown in Fig. 4. We observe from Fig. 4, that IMFs c7 to c10 are above the confidence interval (threshold). The partial reconstruction of IMFs c7 to c10 and residual provides the denoised voltage signal. Fig. 5 shows the comparison between denoised and original voltage signal. The Hilbert-Huang Transform of the IMF components of the noisy signal is shown in Fig. 6.

4.2 Oscillatory Transient

A transient is an undesirable momentary disturbance of the supply voltage caused due to lighting, equipment start-up and shut down, or by welding equipment. A transient can result in processing error, equipment damage, and degradation of electrical insulation. Transients can be classified as *impulsive* or *oscillatory*. An oscillatory transient of duration 20ms superimposed on a noisy voltage signal is shown in Fig. 7. Applying EMD, the signal decomposes into IMFs c1 to c10 and residual component as shown in Fig. 8.

After obtaining the IMFs, the energy model of the noisy transient signal along with the noisy only model are obtained as shown in Fig. 9. From Fig. 9, we observe that the first three IMFs c1 to c3 are below the 99% confidence interval (threshold) and therefore can be regarded as noise. The IMFs c4 to c10 contain oscillatory transient and normal voltage

signals. In Fig. 9, we observe a sudden increase in energy at c4 and c7. Therefore, we partially reconstruct c4 to c6 to
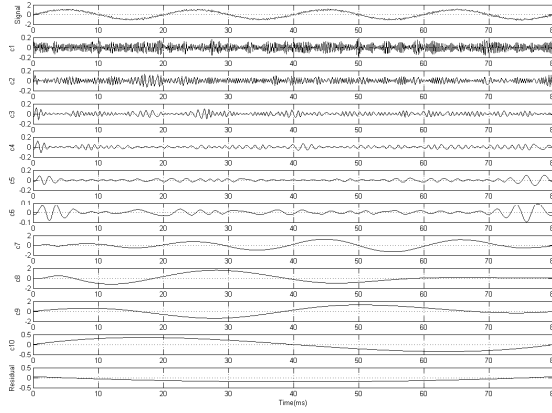
obtain the oscillatory transient signal as shown in Fig. 10 and c7 to
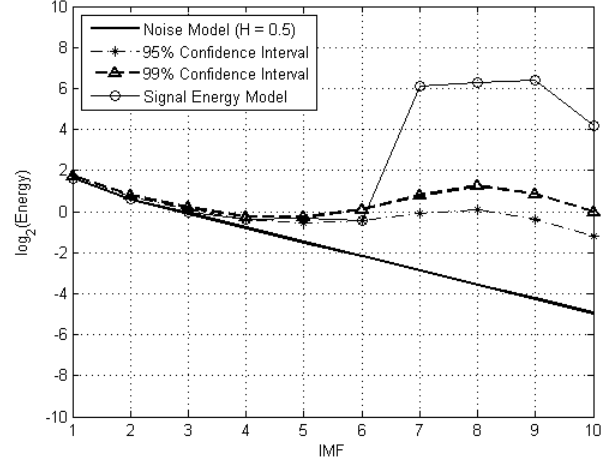

Fig. 3. IMFs and residual component of noisy signal.

Fig. 4. Energy distribution of noisy signal and noise only along with confidence interval with respect to IMF indices.

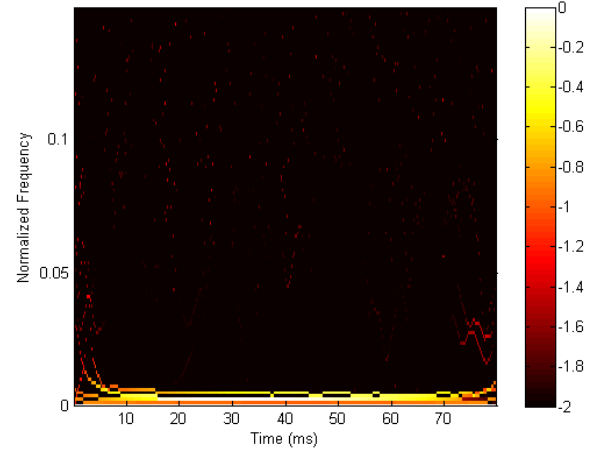Fig. 5. Denoised voltage signal obtained by partial reconstruction of IMFs c7-c10 and residual (top) and original voltage signal (bottom).

Fig. 6. Hilbert-Huang Transformation of the IMFs obtained from noisy voltage signal.

c10, plus the residual component to obtain denoised normal voltage signal as shown in Fig. 11.

The Hilbert-Huang Transform of the noisy transient signal IMFs is shown in Fig. 12. Comparing Fig. 6 and Fig. 12, which show the HHTs of the noisy signal and noisy oscillatory transient signal respectively, we can clearly identify the oscillatory transient disturbance occurring between 50ms and 70ms.

5. Conclusions and Future work

In this paper, we denoise electrical signal and detect power disturbances, such as oscillatory transients, in noisy conditions using EMD and the statistical energy model of the IMFs. The statistical property of fGn is used to obtain noise model. In addition, we demonstrate an approach where the Hilbert-

Huang Transform is used to detect oscillatory transients. Our findings suggest that EMD and the statistical properties of the IMFs can be an effective tool in power quality monitoring.

As a part of future research, we would like to evaluate the performance of EMD in power quality monitoring against other signal processing algorithms, especially the Wavelet Transformation. We would also continue to study the detection of other power disturbances in noisy conditions.
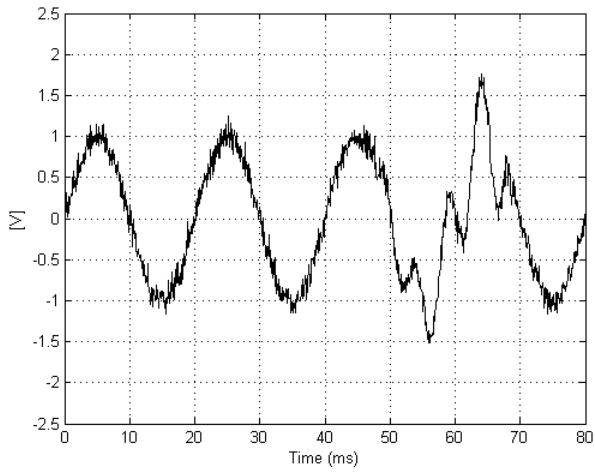
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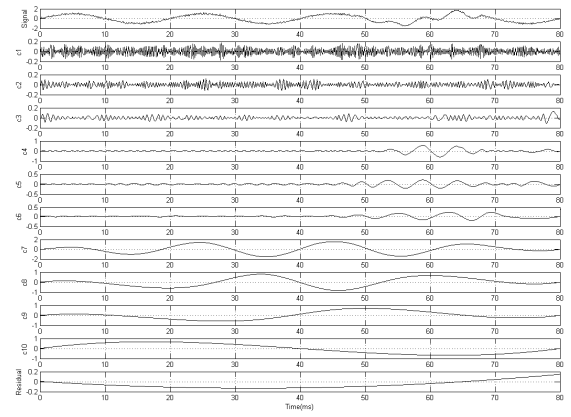Fig. 7. Noisy oscillatory transient voltage signal.

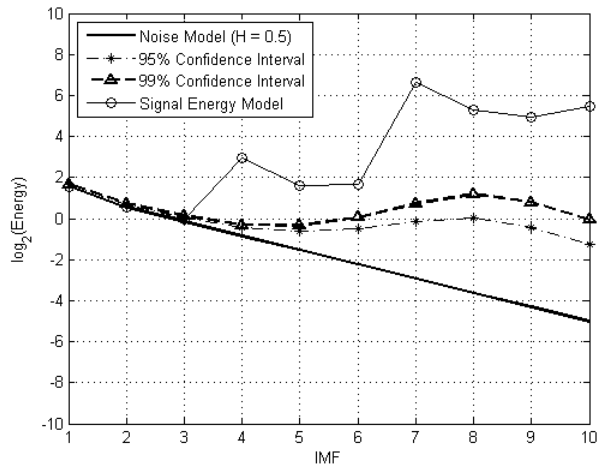Fig. 8. IMFs and residual component of noisy oscillatory transient signal.

Fig. 9. Energy distribution of noisy oscillatory transient signal and noise only along with confidence interval with respect to IMF indices.
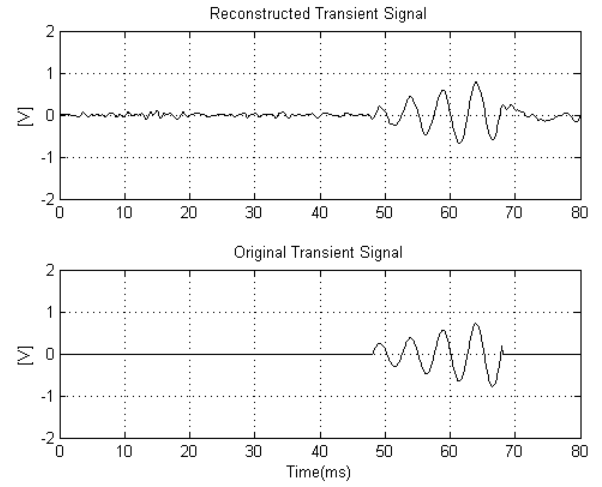

Fig. 10. Oscillatory transient signal obtained by partial reconstruction of IMFs c4 to c6 (top) and actual oscillatory transient signal (bottom).
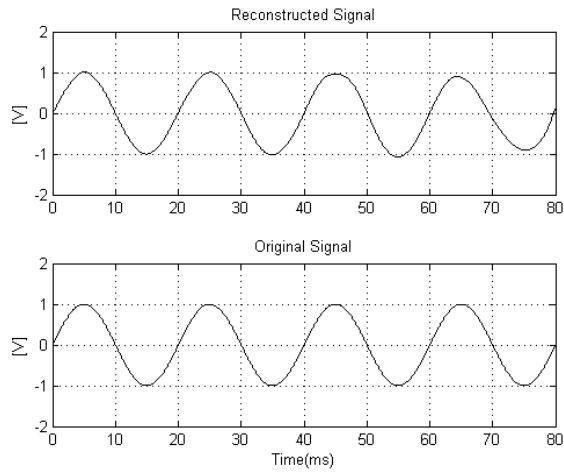

Fig. 11. Denoised voltage signal obtained by partial reconstruction IMFs c7 to c10 and the residual component (top) and normal voltage signal (bottom).
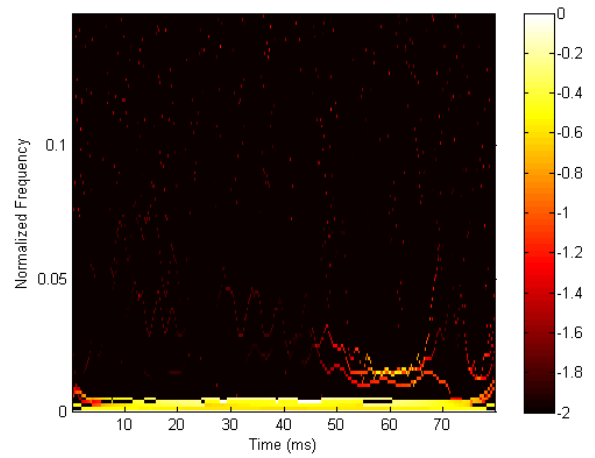

Fig. 12. Hilbert-Huang Transformation of the IMFs obtained from noisy oscillatory transient signal.

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