Analysis of the Mediterranean Precipitation Associated with the North Atlantic Oscillation (NAO) Index via Hilbert-Huang Transformation

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Abstract

The state of the atmosphere is governed by the classical laws of fluid motion and exhibits a great deal of correlations in various spatial and temporal scales. The correlations are crucial to understand short and long-term variability in climate. The most significant correlations are recognized as teleconnection patterns. One of those connections is the North Atlantic Oscillation (NAO) which is associated with anomalous weather patterns in eastern United States, Western Europe and Mediterranean. Although, there is a consensus on physically relationship between the NAO index and precipitation series, the results of statistical analysis drawbacks such strong correlation. The gap between the physical and statistically approach might be filled by integrating through the time of the NAO index and precipitation. After transforming both of the NAO index and precipitation series, each of them behaves as the output of a low-order dynamical system. In this study, Empirical Mode Decomposition (EMD) method of Huang is employed to cut up the total NAO index data into different narrow-band frequency components, and then studying each of the components with a resolution harmonized to its scale of which are called Intrinsic Mode Functions (IMFs) of the underlying physical dynamics having advantages over traditional Fourier analysis in analyzing physical situations, where the signal contains discontinuities and sharp spikes in a domain of non-linear and non-stationary space case. The EMD method is performed to extract oscillatory modes in the total NAO index in order to reduce the relationship between the each of the IMFs and large-scale precipitation comparing with the embedding method of non-linear dynamical systems approach. The results show that the total NAO index exhibits substantial organized and correlated structures with precipitation variability over the Mediterranean.

Keywords: Empirical mode decomposition, Hilbert-Huang Intrinsic Mode Functions, Mediterranean, precipitation, NAO index, organized structure

Introduction

Teleconnection patterns are simultaneous correlations in the fluctuations of the large scale atmospheric parameters at widely different points on the earth (Türkeş and Erlat, 2003, 2005). The effect of these patterns could be significant through the dominant modes of atmospheric variability. Wallace and Gutzler (1981) have surveyed the entire northern hemisphere teleconnections and showed their interactions with the semi-permanent cells of general circulation of atmosphere.

Precipitation could be thought as an output of a discrete system whereas teleconnections are continues. Therefore, to display the interaction of a discrete and a continuum system might be shown by time series analysis techniques; correspondingly, the mathematical assumptions of the methods should be studied very carefully. North Atlantic Oscillation (NAO) indicates the exchange of the atmospheric mass between the Greenland/Iceland and the regions of North Atlantic Ocean between *35-40 ^{o}N* latitudes, and it is characterized by a north-south dipole. To extract the organized-structures embedded in the NAO index could be very informative for illumination the underlying regional precipitation variability. If there is organized-structure then it could be thought that there might be a deterministic rule (predictable) behind the interrelations.

The regional precipitation might be directly tied to the transportation and convergence of synoptic eddy activity associated with the shifts the storm tracks accompanied by the changes in the mean circulation patterns over the North Atlantic (Rogers, 1997). The NAO index is termed as purely stochastic process behaves fully randomly without its organized-structures (Stephenson *et al.,* 2000). However, the results of this study show that NAO could not be mentioned a purely stochastic process. In other words, a characteristic problem in climate system is that involved processes are very complex with very huge number of degrees of freedom. Although each of free components might, at certain time, have crucial role there is a general consensus that many important features of the climate governed by rather low-dimensional dynamical subsystems. The early study of Lorenz (1963), points at this major problem in atmospheric systems. Hence, the major problem is to find the right decomposition for extracting the significant physical signals (organized-structures) in a noisy process. There are several methods to reduce the complex system to the low-order subsystems, such as: Singular Spectrum Analysis (SSA) or Temporal Principal Component Analysis (TPCA), Principal Component Analysis (PCA), Canonical Correlation Analysis (CCA), Principal Interaction and Oscillation Patterns (PIP and POP) and Independent Component Analysis (ICA) (North l984; Broomhead and King, 1986; Preisendorfer, l988; Ghil and Vautard, 1994; Allen and Smith, 1994; von Storch and Zwiers, 1999; Tatli *et al*., 2004, 2005).

The aim of those methods is to reduce the high-dimensional complex system into the understandable simple low-order sub-systems. Empirical Mode Decomposition (EMD) of Huang (Huang *et al.,* l998) is such a method having some successfully applications in atmospheric sciences (Wu *et al.*, 1999; Wang *et al.*, 2000; Ouergli, 2002; Coughlin, 2003; Duffy, 2004). The EMD decomposes the noisy system into the simple narrow-band frequency of subsystems leading to employ the causal Hilbert transformation.

In organization of the paper; in the following section, the employed methods are introduced. The results of this study are given in Section 3, and the study is concluded with an outlook and a summary in Section 4.

Data and Methods

In the paper, the monthly NAO index series are selected according to Hurrell (1995), Jones *et al.* (1997) and Rogers (1997) during the period of 1950-2004. The so called empirical mode decomposition method which is first introduced by Huang *et al.* (1998) is employed. This technique iteratively decomposes a univariate signal into oscillating subsystems. The different components match the original signal itself, and the time-frequency representation extracted from the decomposition is called the Hilbert amplitude spectrum. The principle of this technique is to decompose a univariate signal *x(t)* into a arithmetic sum of intrinsic mode functions that:

1) Each mode may or may not be linear, and will have the same number of extreme and zero-crossings,
2) IMFs are symmetric with respect to the “local mean”, and are obtained using the “sifting” algorithm of Huang *et at.* (l998).

With this definition the local mean, the modes of different time scales can be separated by their characteristic scales, defined as the time lapses between the successive extreme. Once separately each mode should be independent of the others; they have no multiple extreme between successive zero-crossings (Huang et al., l998). This assumption requires that the extracted intrinsic functions being regular by the sense of organized-structure of the time evaluation of the dynamical systems (Fraedrich, 1986; Rabinovich and Thieberger, 1987). The algorithm of Huang *et al.* (1998) is a sifting empirical process having the resemble logic of the time-lagged shifting process (Fraedrich, 1986; Parker and Chua, 1980), but it has not a closed form; could be briefly explained as follows:

1) The number of local extreme of intrinsic functions and its zero-crossings must either be equal or different-at most by one.
2) At any time *t*, the mean value of the “upper envelope” (determined by the local maxima) and the “lower envelope” (determined by the local minima) is zero,
3) If the data were totally devoid of extreme but contained only inflection points, then it can be differentiated once or more times to reveal the extreme,
4) The final results can be obtained by integration of the components.

Let us explain the algorithm given above mathematically; an univariate *x(t)* signal has its Hilbert transformation *h(t)* as in the following.

$$h(t) = \frac{1}{\pi} P \int_{-\infty}^{+\infty} \frac{x(t')}{t - t'} dt' \tag{1}$$

where *P* indicates the Cauchy principal value. The analytic form (Hilbert or complex) of the signal, *z(t),* is then obtained as:

$$z(t) = x(t) + ih(t) = a(t)e^{i\theta(t)} \quad (2)$$

Herein *a(t)* represents the amplitude and $\theta(t)$ is the phase angle of the signal which is defined as:

$$a(t) = \left[x(t)^2 + h(t)^2\right]^{1/2}; \;\; \theta(t) = \arctan\left(\frac{h(t)}{x(t)}\right) \quad (3)$$

where *a(t)* represents the slow component, and $\theta(t)$ indicates fast component. Additionally, the instantaneous frequency is defined as:

$$w(t) = \frac{d\theta(t)}{dt} \quad (4)$$

The instantaneous frequencies are obtained by applying the Hilbert transform to the functions of the above type which are very well localized in the time-frequency domain and reveals the important characteristics of the signal. If we denote the intrinsic functions by *imf_i(i =1, .., m)* and the residual by *res*, respectively; then the original signal can be illustrated by

$$x(t) = \sum_{i=1}^{m} imf\__i(t) + res(t) \quad (5)$$

where *res(t)* generally indicates the mean (or the trend) embedded in the signal, and *m* represents the number of the intrinsic functions. The major difference between Fourier and EMD is that EMD uses “narrow-band” while Fourier approach is “mono-component”

The large-scale precipitation rate data set is obtained from NCEP/NCAR reanalysis data sets (*Kalnay et al*., 1986; Kistler *et al*., 2001) between the range of 30-60 ^{o}N and 20-45 ^{o}E during the period of 1950-2004. In first step, the correlation coefficients between the NAO index and gridded precipitation rate series are obtained in order to underline if there is a significant relation between them. As a result of the correlation analysis, it is found that there is not strong correlation between the NAO index and precipitation series.

Intuitionally, there might be involved strong relation between the NAO index and precipitation series; therefore the NAO index is decomposed by EMD method to extract the significant empirical modes of the NAO index which of them might be correlated with the precipitation rate series. The discussion of this process is presented in the next section of the paper.

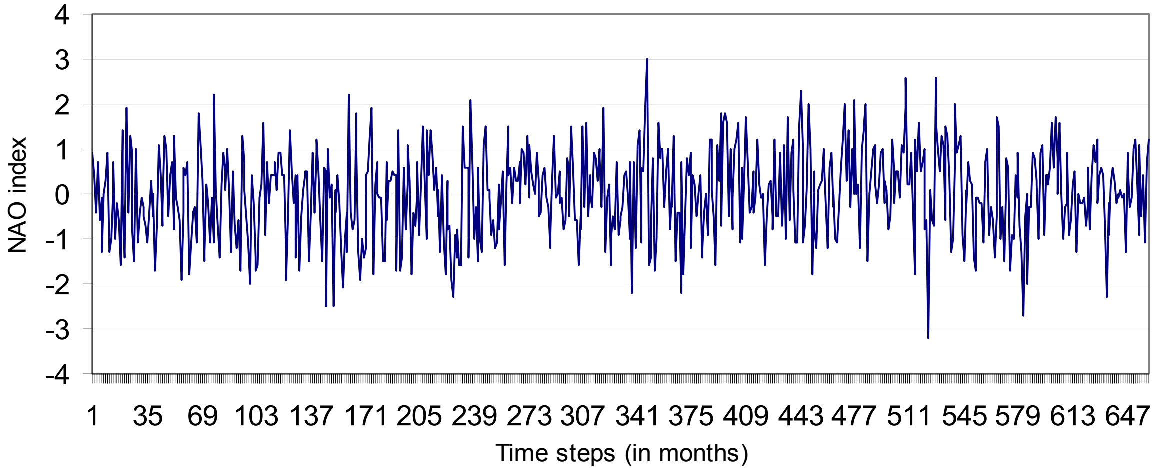
Secondly, the cross-integrated NAO index (named as total NAO index) is found by the cumulative summing through the time, and then examining embedding dimensions in the total NAO index by the False Nearest Neighbors method of Kennel *et al.* (1992). The False Nearest Neighbors dimension is a method of choosing the minimum embedding dimension of a one-dimensional time series. This method finds the nearest neighbor of every point in a given dimension, then checks to see if these are still close neighbors in one higher dimension. The percentage of False Nearest Neighbors should drop to zero when the appropriate embedding dimension is reached. The appropriate embedding dimension is a bit of a judgment call. If the number of false nearest neighbors is not zero but is very small, the embedding dimension may still be suitable for most forms of analysis (for detail information, see the paper of Kennel *et al.* (1992)).

Intuitionally, the precipitation process is indeed a discrete process, to convert the discrete precipitation process into a continuum frame; a similar way is followed while obtaining the total NAO index, the monthly precipitation rate series are cumulative summed through the time after they are scaled being in the standardized series (has zero mean and variance one). Now the nature of the total NAO index and total precipitation series might be so called that they are self consistent with each other.

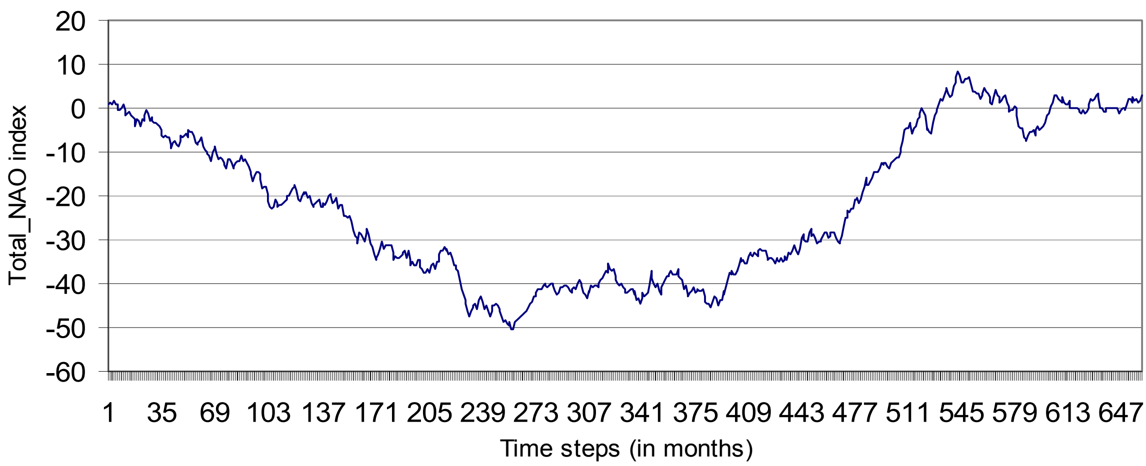
Precipitation process is a very important component of a water resources system, which is the one of the input components of the system. Additionally, the precipitation component is observable but not be controllable. Hence, the temporal behavior of this component can be mentioned vitally for the water resources systems. On the other hand, NAO is an important atmospheric oscillation process affects changes and variability of precipitation, *i.e.,* it has indirectly effects on the continuum and input of the water resources systems over a large area of Europe and the Mediterranean regions.

Application and Results

The plots of the NAO and total NAO indices are shown in Figure 1. According to Figure 1, NAO index is assumed a high dimensional system (it might be named a random walk), whereas the total NAO index behaves as a low-order dynamical system. In order to clarify our intuitional assumptions; the probability mass functions (histograms) of the both NAO and total NAO indices are extracted as shown in Figure 2.



(a)

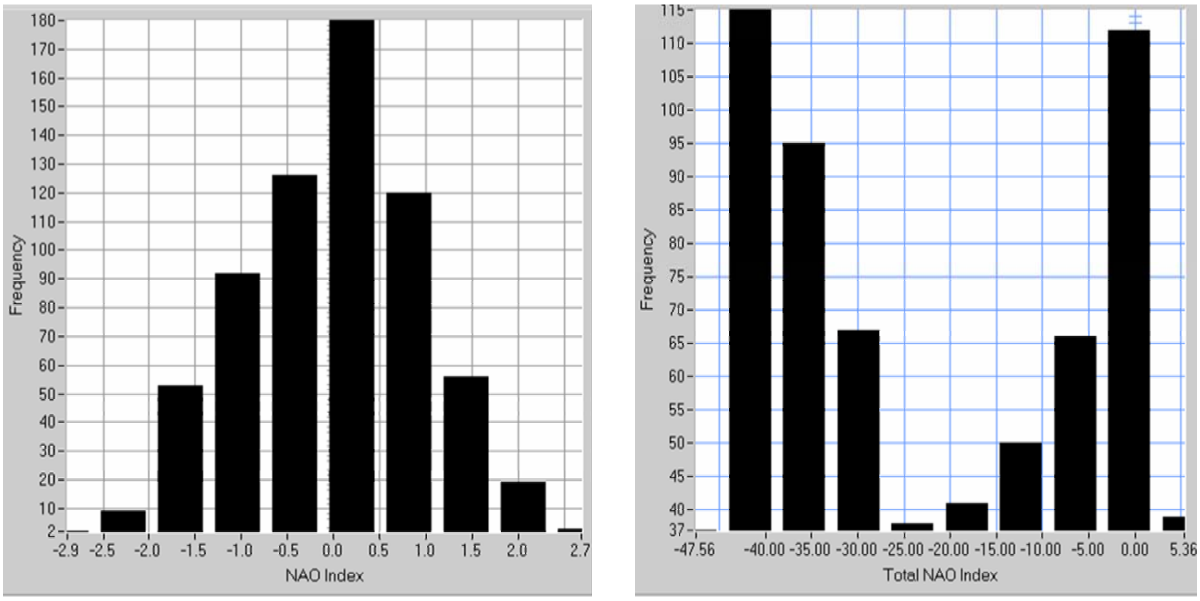


(b)

Figure 1. NAO indices: (a) NAO index, (b) Total NAO index

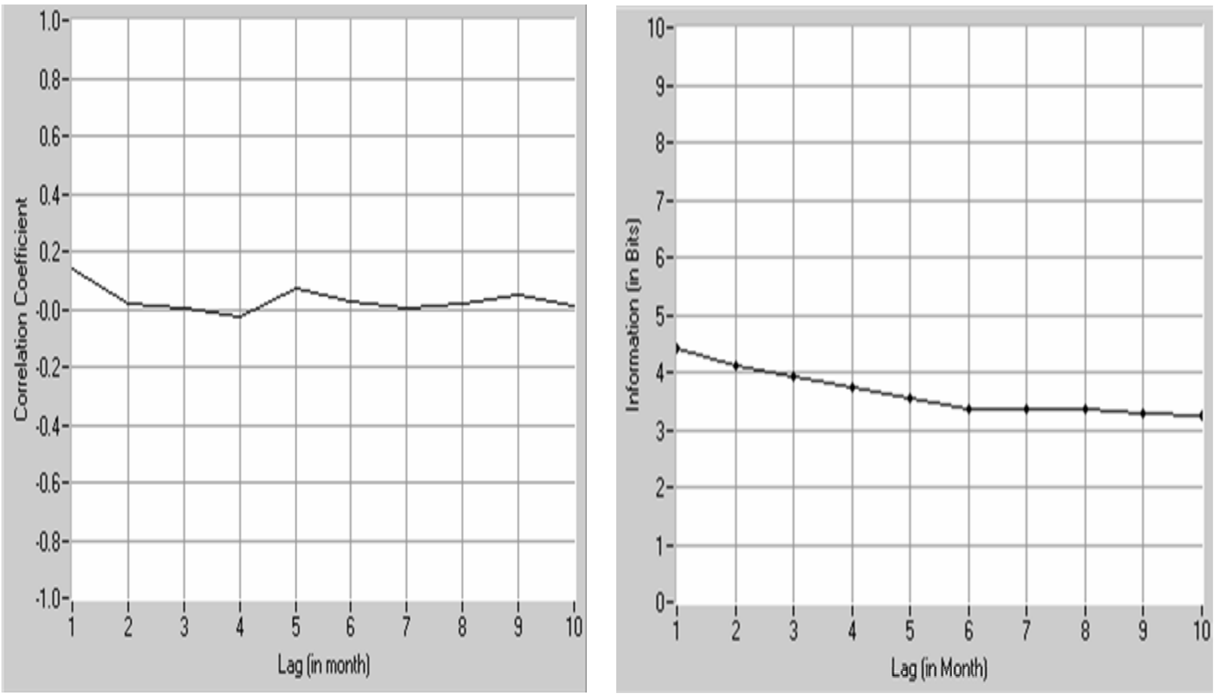
The correlation functions of the NAO indices are shown in Figure 3. The correlation function of total NAO index is computed by using mutual information function, since it has highly non-stationary properties (Figure 1 b) that classical Pearson correlation function might not show the general characteristics of its temporal behavior (*i.e.,* variability). As seen in Figure 3, total NAO index has a smooth correlation function comparing with the correlation function of the NAO index which indicates that it has low dimensional characteristics. To improve our intuition, the method of the False Nearest Neighbors (Kennel *et al,* 1992) is employed by the aid of the Fortran codes of TISEAN-software of Kantz and Schreiber (1997); the results are shown in Figure 4.

To detect the number of embedding dimensions in the NAO index, the same method is also applied to the NAO index, and the results are shown in Figure 5. According to the Figure 5, we can not make an assumption whether the NAO index is produced from a dynamical system; it might be called a high dimensional stochastic process. In addition, there are breaking points while matching to the true embedding dimension between the first and third dimensions. In words, the NAO index may be forced by some noisy processes whereas the total NAO index shows very smooth-characteristics.



(a) (b)

Figure 2. The histograms of NAO indices: (a) NAO index; (b) Total NAO index. The total NAO index indicates the sign significance of NAO that it is in a bimodal form.



(a) (b)

Figure 3. Temporal behavior of the NAO indices: (a) NAO index; (b) Total NAO index

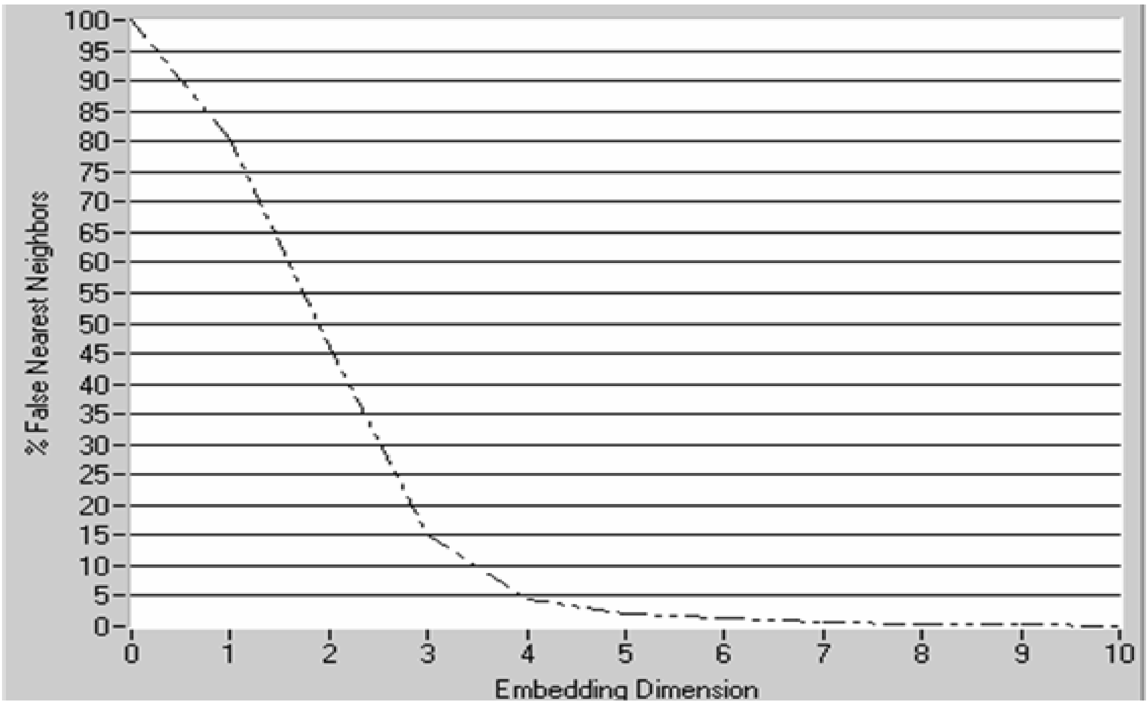


Figure 4. Extracting the significant number of the embedding dimensions in the total NAO index. The number of the significant dimension might be accepted between *4* and *5* with 5% statistical error.

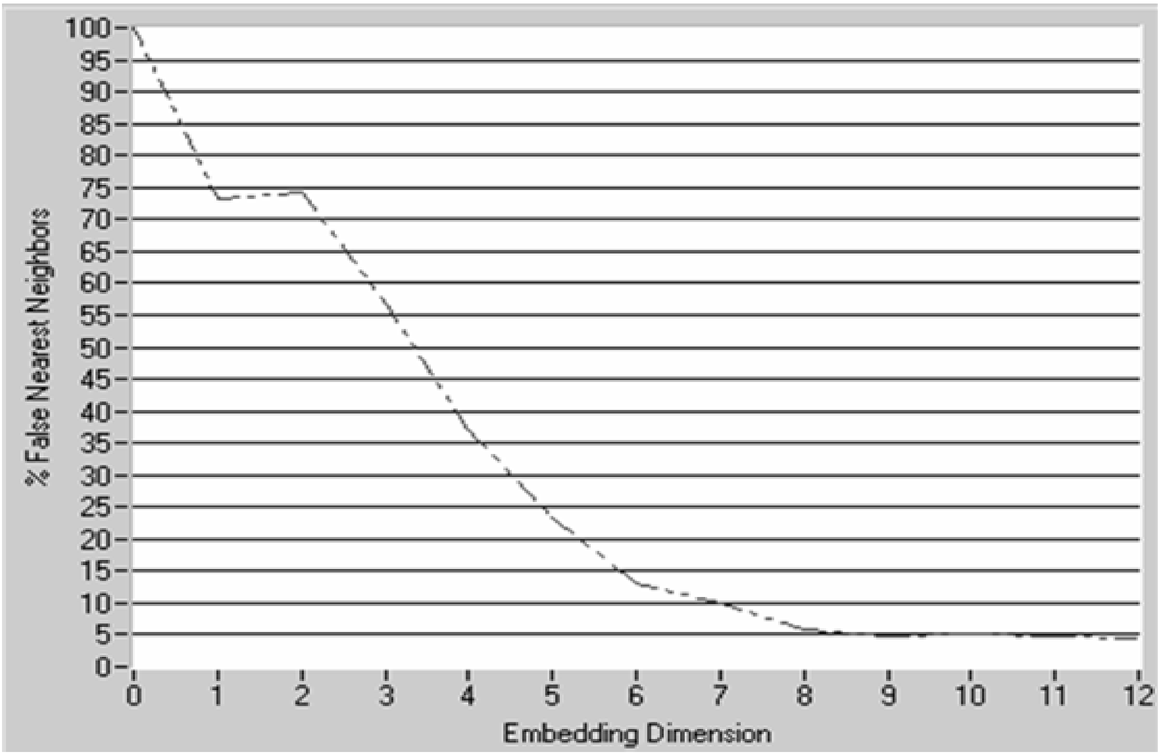
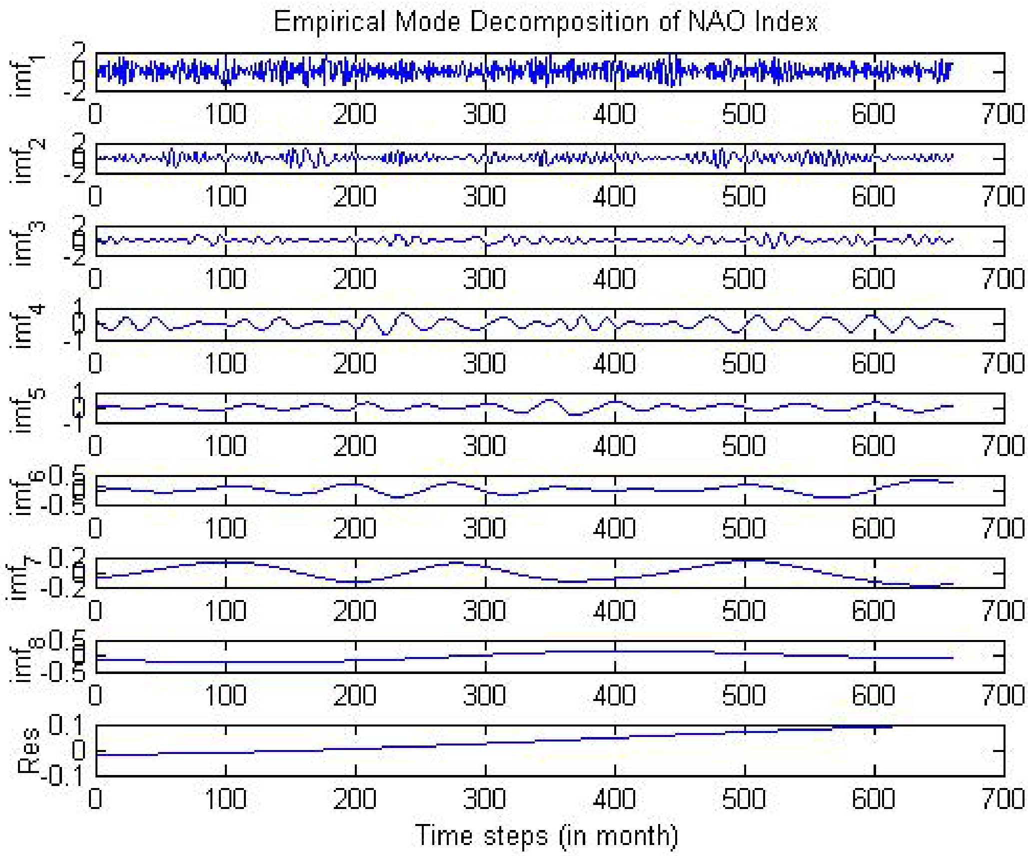
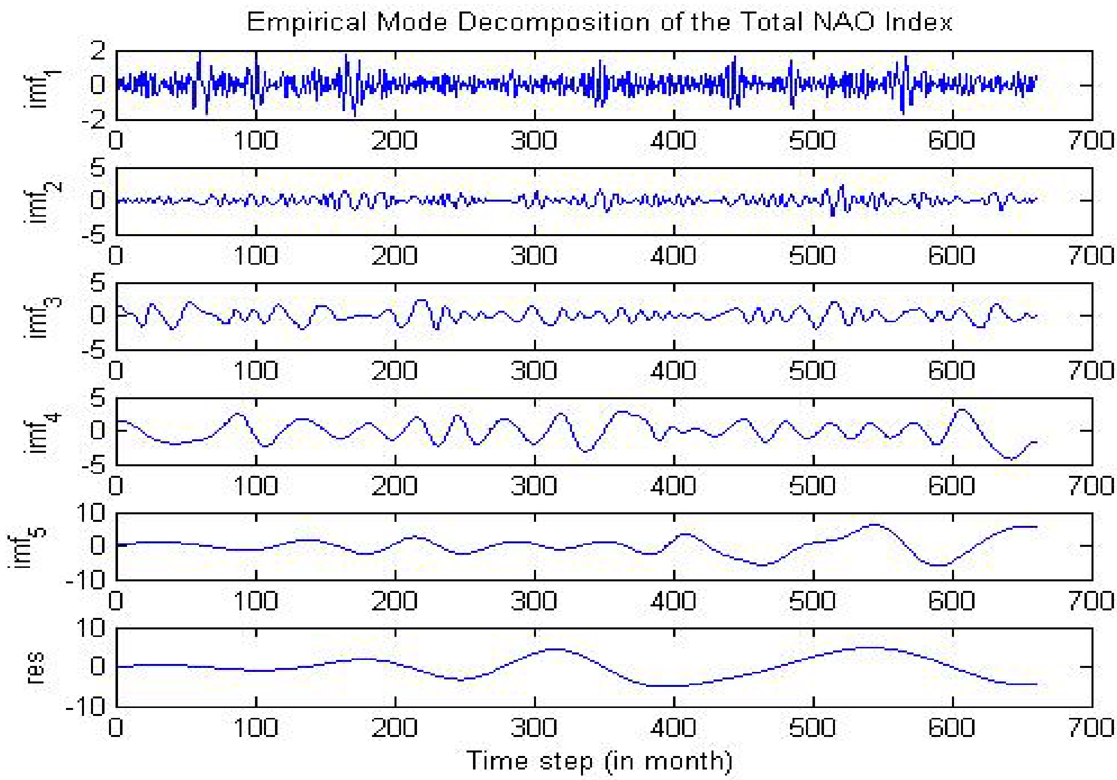


Figure 5. Detection the number of significant embedding dimensions in NAO index. It behaves like a high-order random walk process.



(a)



(b)

Figure 6. IMFs of the NAO indices. (a) NAO index; (b) Total NAO index.

The extracted intrinsic mode functions of the NAO indices are shown in Figure 6. There are nine oscillatory modes of the NAO index as expected due to our assumptions, whereas there are only six significant oscillatory modes in the total NAO index. These results support our intuitional assumptions.

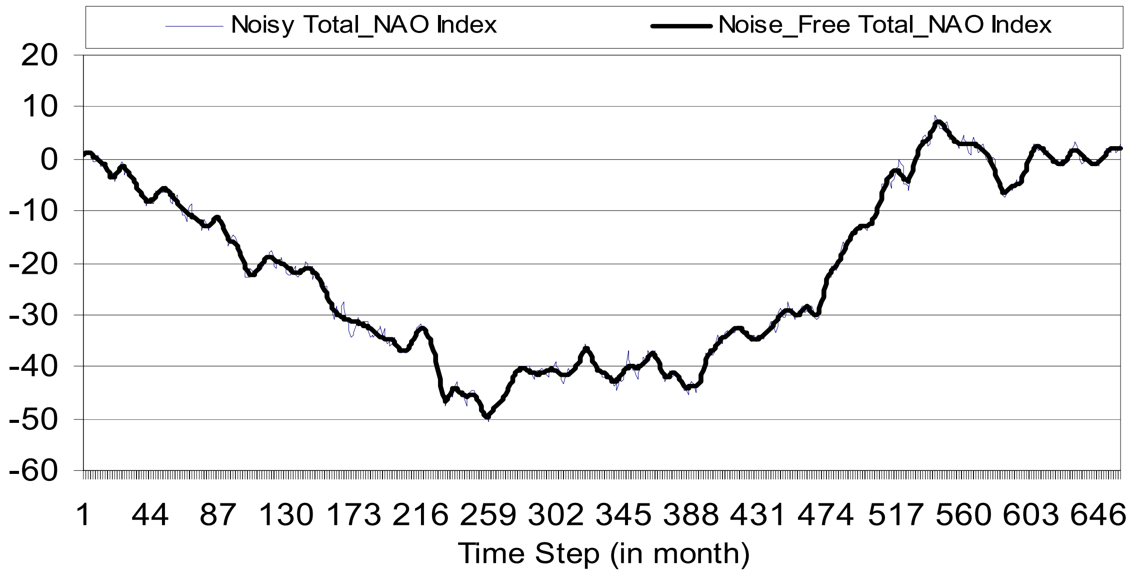


Figure 7. Total NAO index after EMD process.

The first two IMFs of the total NAO index behave noisy; therefore they might be termed as blue-noises or unpredictable components by both the deterministic (mathematical) and statistical methods. Hence, these two components are eliminated in the total NAO index. The results of the last four IMFs of the total NAO index are now handled as the significant oscillatory modes which are consistent with the result of the method of the False Nearest Neighbors as shown in Figure 4. The arithmetic summation of these four IMFs is now our noise-free total NAO index as shown in Figure 7.

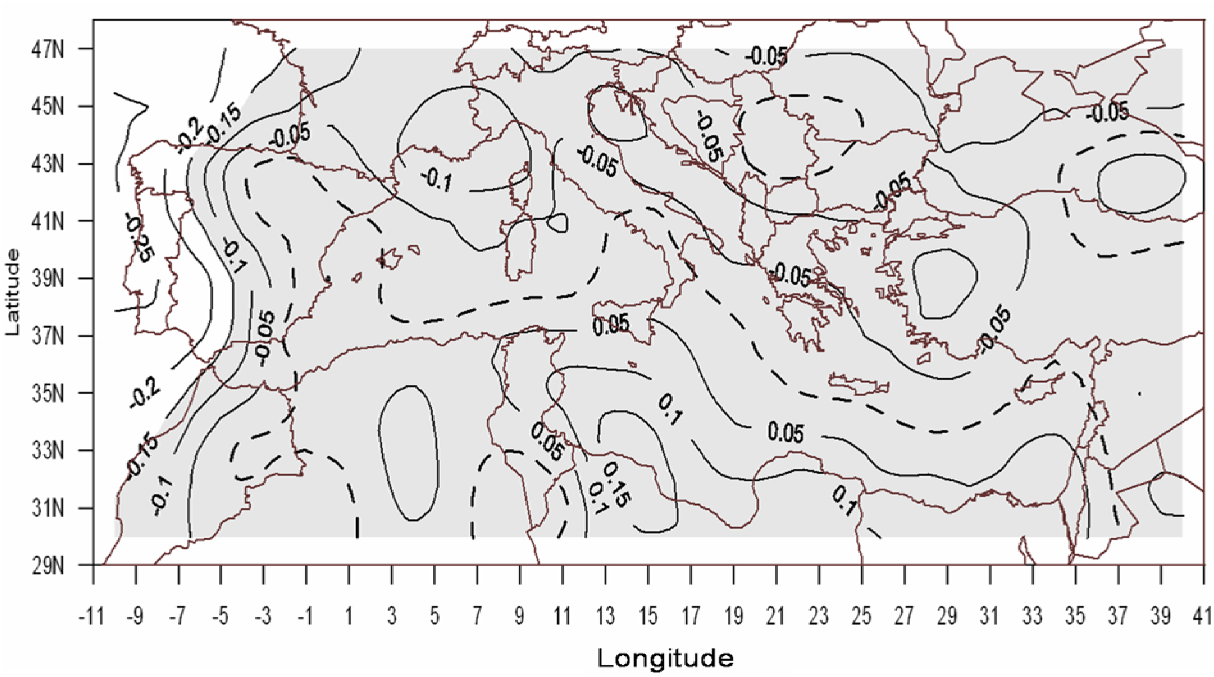


Figure 8. Spatial distribution of correlation coefficients between the precipitation rate and NAO index series. The lightened regions represent the significant correlation coefficients based on 90% significant level.

The scientific contribution our approach may be queried because the total NAO index might not be, in fact, the product of the North Atlantic Oscillation. The total NAO index is just termed by a very simple way of integration of the NAO index in the study. In other words, the NAO index might be seen as the derivative (difference) of the total NAO index which shows the temporal effects of the physical characteristics of North Atlantic Oscillation.

On the other hand, according to the literature, with the aid of original observations, the NAO must affect to the precipitation process, and this should also be involved by statistical analysis (see *e.g.,* Türkeş and Erlat, 2003, 2005). However a statistical analysis can not fully show the significant relationship; may be during the winter season, there might be significant but not stronger correlations which are exhibited in some regions of Mediterranean. The results of the correlation analysis between the NAO index and precipitation rate series are given in Figure 8. According to this Figure, there are not strong correlation fields as we discussed early; however the results of the total NAO index approach (Figure 9) is more informative than the using the raw NAO index given in Figure 8.

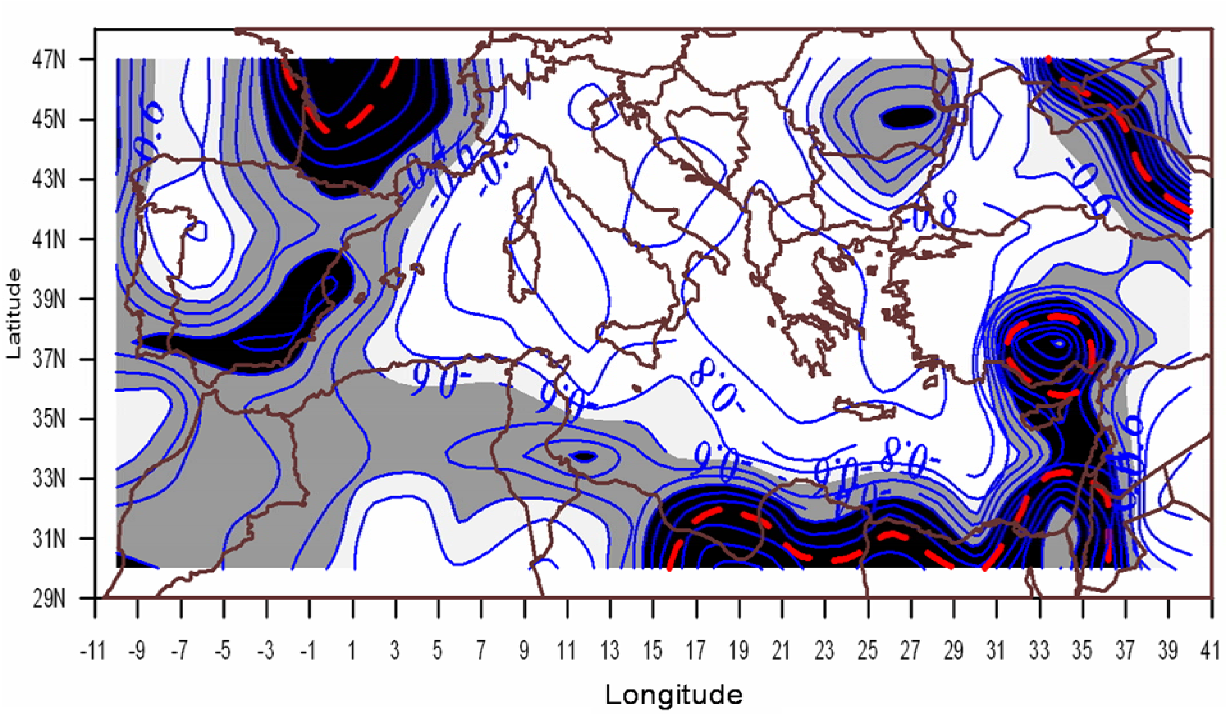


Figure 9. The correlation map between the total precipitation rate and total NAO index. The lightened regions represent the correlation coefficients which are greater and equal to *0.7*, those are accepted being at the predictable level for a traditional linear model (regression).

Conclusions

In this study, the temporal behavior the NAO is investigated by taking its integration through the time, and decomposed by the EMD method of Huang (1998). The results of the proposed approach show that a noisy, non-stationary and non-linear high-order complex process (Stephenson *et al.,* 2000) might be decomposed into its simple, but understandable sub-structures. The study also proposes to manipulate the structure of the probability mass functions (histograms), and the False Nearest Neighbors method of Kennel *et at.* (1992); in order to check our assumptions while extracting the organized-qualitative knowledge in the complex North Atlantic Oscillation process. The proposed approach does not destroy the information content in the NAO index, but smoothing it, besides reconstruction the process can be succeeded with taking the time derivate (difference) of the total NAO index.

The open point of this study may be seen in using of the tools of Embedology (Sauer *et al*., 1991). Since these tools require very long records; even if the length of the NAO index is not sufficient long for employing the tools of Embedology, the results of this study may be seen a tool for practical applications. Another aim of this study is to bridge the dynamical systems and statistical approaches. An important result might be underlined that although the NAO index shows fully noisy structure, the organized sub-components of it have slow low-order structures which of them might lead to improve the prediction framework of the effects of the North Atlantic Oscillation on the water resources systems.

References

Allen, M. R., L. A. Smith, 1994: Investigating the origins and significance of low frequency modes of climate variability. *Geophys. Res. Let.*, **21**, 883-886.

Broomhead, D. S., G. P. King, I986: Extracting qualitative dynamics from experimental data. *Physica D*, **20**, 2 l 7-236.

Coughlin, K. T., 2003: Stratospheric and Troposheric Signals Extracted Using Empirical Mode Decomposition. *PhD Thesis*. Applied Mathematics, University of Washington.USA.

Duffy, D. D., 2004: The application of Hilbert-Huang transforms to meteorological datasets. *J. Atmos. and Ocean. Tech.*, **21**, 599-61 l.

Fraedrich, K., 1986: Estimating the dimensions of weather and climate attractors. *J. Atmos. Sci.*, **43**, 419–432.

Ghil, M., R. Vautard, 1994: Interdecadal oscillations and the warming trend in global temperature series. *Nature*, **310**, 324-327.

Huang, N. E., Z. Shen, S.R. Long, M. C. Wu, H. H. Shih, Q. Zheng, N. C. Yen, C.C. Tung, H. H. Liu, 1998: The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. *Proc. The Royal Soc. London (A),* **454**, 903-995.

Hurrell, J. W., 1995: Decadal trends in the North Atlantic Oscillation: regional temperatures and precipitation. *Science*, **269**, 676-679.

Jones, P. D., T. Jónsson, D. Wheeler, 1997: Extension to the North Atlantic Oscillation using early instrumental pressure observations from Gibraltar and South-West Iceland. *Int. J. Climatol.*, **17**, 1433-1450.

Kalnay, E., M. Kanamitsu, R. Kistler, W. Collins, D. Deaven, L. Gandin, M. Iredell, S. Saha, G. White, J. Woollen, Y. Zhu, M. Chelliah, W. Ebisuzaki, W. Higgins, J. Janowiak, K. C. Mo, C. Ropelewski, J. Wang, A. Leetmaa, R. Reynolds, R. Jenne, D. Joseph, 1996: The NMC/NCAR 40-Year Reanalysis Project. *Bull. Amer. Meteor. Soc.*, **77**, 437-471.

Kantz, H., T. Schreiber, 1997: Nonlinear Time Series Analysis. Cambridge University Press, Cambridge.

Kennel, M. B., R. Brown, H. D. I. Abarbanel, 1992: Determining embedding dimension for phase-space reconstruction using a geometrical construction. *Phys. Rev.A*, **45**, 3403-3411.

Kistler, R., E. Kalnay, W. Collins, S. Saha, G. White, J. Woollen, M. Chelliah, W. Ebisuzaki, M. Kanamitsu, V. Kousky, H. Van Den Dool, R. Jenne, M. Fiorino, 2001: The NCEP-NCAR 50-Year Reanalysis: Monthly Means CD-ROM and Documentation. *Bull. Amer. Meteor. Soc.,* **82**, 247-268.

Lorenz, E. N., 1963: Deterministic nonperiodic flow. *J. Atmos. Sci.,* **20**, 130-141.

North, G., 1984: Empirical orthogonal functions and normal modes. *J. Atmos. Sci.,* **41**, 879- 887.

Ouergli, A., 2002: Hilbert transform from wavelet analysis to extract the envelope of an atmospheric mode: examples. *J. Atmos. and Ocean. Tech.*, **19**, 1082- 1086.

Parker, T. S., L. O. Chua, 1989: Practical Numerical Algorithms for Chaotic Systems. Springer-Verlag.

Preisendorfer, R. W., 1998: Principal Component Analysis in Meteorology and Oceanography. Elsevier.

Rabinovich, A., R. Thieberger, 1987: Time series analysis of chaotic signals. *Physica D*, **28**, 409-415.

Rogers, J. C., 1997: North Atlantic storm track variability and its association to the North Atlantic Oscillation and climate variability of Northern Europe. *J. Climate*, **10**, 1635-1647.

Sauer, T., J. A. Yorke, M. Casdagli, 1991: Embedology. *J. Stats. Phys.*, **65**, 579-616.

Stephenson, D. B., V. Pavan, R. Bojariu, 2000: Is the North Atlantic Oscillation a random walk? *Int. J. Climatol.*, **20**, 1-18.

Tatli, H., H. N. Dalfes, S. S. Mentes, 2004: A statistical downscaling method for monthly total precipitation over Turkey. *Int. J. Climatol.*, **24**, 16 l -180.

Tatli, H., H. N. Dalfes, S. S. Mentes, 2005: Surface air temperature variability over Turkey and its connection to the large-scale upper air circulation via multivariate techniques. *Int. J. Climatol.*, **25**, 331-350.

Türkeş, M., E. Erlat, 2003: Precipitation changes and variability in Turkey linked to the North Atlantic Oscillation during the period 1930-2000. *Int. J. Climatol.*, **23**, 1771-1796.

Türkeş, M., E. Erlat, 2005: Climatological responses of winter precipitation in Turkey to variability of the North Atlantic Oscillation during the period 1930-2001. *Theor. and App. Climatol.*, **81**, 45-69.

Von Storch, H., F. W. Zwiers, l999: Statistical Analysis in Climate Research. Springer/ Cambridge University Press, Cambridge, UK.

Wallace, J. M., D. S. Gutzler, 1981: Teleconnections in the geopotential height field during the Northern Hemisphere winter. *Mon. Wea. Rev.,* **109**, 784-812.

Wang, J., C. S. Chern, A. K. Liu, 2000: The wavelet empirical orthogonal function and its application to the analysis of internal tides. *J. Atmos. and Ocean. Tech.*, **17**, 403- 1420.

Wu, M. L., S. Schubert, N. E. Huang, 1999: The development of the south Asian summer Monsoon and the intraseasonal oscillation. *J. Climate*, **12**, 2054-2075.