Intermittency study of river flow fluctuations by using arbitrary order Hilbert spectral analysis

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ABSTRACT: In this paper we present the analysis of a long (32 years) time series of daily river flow data, recorded in the Seine River (France). We apply on this time series a new data analysis technique coming from turbulence studies and time series analysis, namely arbitrary order Hilbert spectral analysis. The latter is an extended version of the Hilbert-Huang transform we developed in order to study the intermittency of time series. This new method provides a scale-frequency decomposition of the original time series, giving a joint pdf $p(\omega, A)$. When marginal moments of the amplitude are computed, one obtains an intermittency study in the frequency space. Applied to river flow data, this shows the scaling range and characterizes the intermittent fluctuations over the range of scales from 4.5 to 60 days.

1 INTRODUCTION

A better understanding of river flow fluctuations is of sharp practical importance, for ecosystem studies (transport properties), for flood understanding and forecasting etc. River flows fluctuate on many scales, see Figure 1: at small scales, river turbulence induces stochastic fluctuations and at larger scales (months or seasons) the river flow fluctuations are the result of complex nonlinear interactions between rainfall processes, topography evaporation and groundawater storage in the river basin. Daily river flow time series thus show fluctuations possessing stochastic properties, as well as deterministic forcing resulting from seasonal or annual meteorological and climatic cycles. Since Hurst (Hurst, 1950) introduced his famous R/S analysis, people have used different methods, for example, correlation analysis, wavelet analysis, detrended fluctuation analysis (Pandey et al. 1998, Hu et al. 2001, Koscielny-Bunde et al. 2006), etc. to characterize the long term memory effects and multifractal properties in the different records of river flow.

Hilbert-Huang transform (or Empirical Mode Decomposition called by other authors) is a method built to deal with nonlinear and nonstationary time series (Huang et al. 1998, 1999, Flandrin and Gonçcalvès, 2004). In this paper, we will present an extended version of Hilbert spectral analysis, namely arbitrary order Hilbert spectral analysis, which can reduce the nonstationary and nonlinear effects, to characterize the intermittency properties of Seine river flow discharge time series.

2 THE DATA: FLOW DISCHARGE OF SEINE RIVER

Seine river is the third biggest river in France. The flow data is provided by the Service de Navigation de la Seine (SNS). This corresponds to daily flow data Q (in m^3/s), recorded from 1 January 1976 to 28 April 2008. There are 11,828 data values, with some missing values due to

some interruptions, for maintenance or because of the failure of measuring devices. The data are displayed in Figure 1, showing some large fluctuations at all scales. The mean and standard deviation of the discharge are 488m^3/s and 349m^3/s. These illustrate the complex and stochastic behavior, with a visible strong annual cycle.

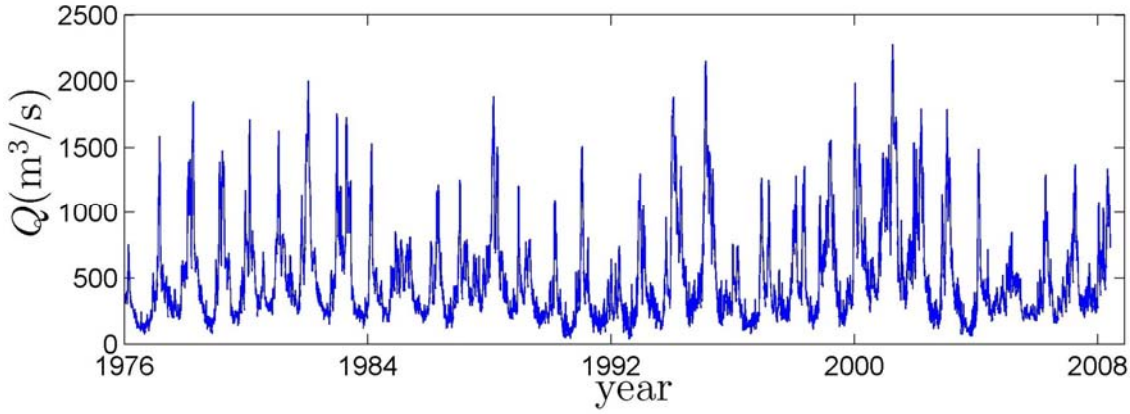

Figure 1. The river flow discharge time series of Seine river, illustrating a clear annual cycle with huge turbulence-like fluctuations at smaller scales.

3 METHODS

3.1 *Hilbert-Huang Transform*

Hilbert-Huang transform is a new method to analyze nonlinear and nonstationary time series, which was developed and proposed by N.E. Huang (Huang et al. 1998, 1999). It contains the following two steps: Empirical Mode Decomposition (EMD) and Hilbert Spectral Analysis (HSA). The starting point of its first step, EMD, is to consider the real time series as multi-components at a very local level, and only mono-component signal has clearly physical meaning and well-behaved Hilbert transform properties. Then Intrinsic Mode Function (hereafter IMF) is used to approximate the mono-component signal, which describes as: a) the number of extrema and zero-crossing must equal or at most differ by one, b) at any point, the mean value of envelopes defined by the local maxima and the local minima is zero. EMD algorithm is then proposed to extract these IMFs

1 The local extrema of the signal $x(t)$ are identified,
2 The local maxima are connected together forming an upper envelope $e_{\max}(t)$, which is obtained by a cubic spline interpolation. The same is done for local minima, providing a lower envelope $e_{\min}(t)$,
3 The mean is defined as $m_1(t) = (e_{\max}(t) + e_{\min}(t))/2$,
4 The mean is subtracted from the signal, providing the local detail $h_1(t) = x(t) - m_1(t)$,
5 The component $h_1(t)$ is then examined to check whether it satisfies the conditions to be an IMF. If yes, it is considered as the first IMF and denoted $C_1(t) = h_1(t)$. It is subtracted from the original signal and the first residual, $r_1(t) = x(t) - C_1(t)$ is taken as the new series in step 1. If $h_1(t)$ is not an IMF, a procedure called "sifting process" is applied as many times as needed to obtain an IMF. In the sifting process, $h_1(t)$ is considered as the new data, the local extrema are estimated, lower and upper envelopes are formed and their mean is denoted $m_{11}(t)$. This mean is subtracted from $h_1(t)$ providing $h_{11}(t) = h_1(t) - m_{11}(t)$. Then it is checked whether $h_{11}(t)$ is an IMF. If not, the sifting process is repeated, until the component $h_{1k}(t)$ satisfies the IMF conditions. Then the first IMF is $C_1(t) = h_{1k}(t)$ and the residual $r_1(t) = x(t) - C_1(t)$ is taken as the new series in step 1.

In practice the above shifting process should be given some proper stoppage criteria, which will guarantee to extract the components with more physical meaning, see Huang et al. (1998, 1999).

After decomposition, the original time series can be represented as superposing by IMFs with one residual:

$$x(t) = \sum_{i=1}^{N} C_i(t) + r_N(t) \tag{1}$$

In its second step, Hilbert spectra analysis (Cohen 1995, Long et al. 1995, Huang et al. 1998), is applied to each IMF:

$$C_i^H(t) = \frac{1}{\pi} P \int \frac{C_1(t')}{t - t'} dt' \tag{2}$$

where P means the principle value. Then we can have an analytic signal:

$$C_i^A(t) = C_i(t) + jC_i^H(t) = A_i(t) e^{j\theta_i(t)} \tag{3}$$

where the amplitude $A_i(t)$ and phase function $\theta_i(t)$ are defined as $(C_i^2(t) + C_i^{H2}(t))^{1/2}$ and $\tan^{-1}(C_i^A(t) / C_i(t))$, respectively. The original signal is finally represented as:

$$x(t) = RP \sum_{i=1}^{N} A_i(t) e^{j\int_0^t \omega_i(t')dt'} + r_N(t) \tag{4}$$

where RP means real part and $\omega_i(t)$ is the instantaneous frequency, which is written as $d\theta_i(t)/dt$. Comparison with the Fourier transform, equation (4) can be regarded as a generalization of Fourier transform: it allows a frequency and amplitude-modulation simultaneously (Huang et al. 1998, 1999). Hilbert spectrum is therefore defined as the square amplitude $H(\omega,t) = A^2(\omega,t)$, which represents the original signal in energy-time-frequency space at very local level. Hilbert marginal spectrum is also defined as:

$$h(\omega) = \int_0^\infty H(\omega,t) dt \tag{5}$$

It is analogous to the FFT power spectrum (Huang et al.1998).

It has been shown that Hilbert-Huang transform is robust with different stopping criteria (Huang et al. 2003). We then just summarize here some advantages of the present method. (1) completely self-adaptiveness or fully data-driven, there is no basis assumptions before the decomposition, (2) very local ability, in other words there is no uncertainty principle limitation, (3) intuitive, see (Huang et al. 1998, 1999, 2003, Flandrin and Gonçcalvès, 2004). Since its introduction, this method has attracted a large interest. It was shown to be an efficient method to separate a signal into a trend and small scale fluctuations on a dyadic bank (Wu and Huang, 2004, Flandrin and Gonçcalvès, 2004), it has also been applied to many different fields (Jánosi and Müller, 2005, Loutridis, 2005, Solé et al. 2007, Schmitt et al. 2007, Su et al. 2008).

3.2 *Arbitrary Order Hilbert Spectral Analysis*

Since the Hilbert spectrum provides very local level representation of the original signal, we then can define the joint probability density function (pdf) $p(\omega, A)$ of frequency $[\omega_i(t)]$ and amplitude $[A_i(t)]$ (Long et al. 1995), which are extracted from all modes $i = 1 \cdots N$ together. The Hilbert marginal spectrum is then rewritten as:

$$h(\omega) = \int_0^\infty p(\omega, A) A^2 dA \tag{6}$$

One can find that the above definition is just a second statistical moment. We then naturally generalize equation (6) into arbitrary order one, which is written as:

$$L(\omega) = \int_0^\infty p(\omega, A) A^q dA \approx \omega^{-\xi(q)} \tag{7}$$

Where $q \geq 0$, and $\xi(q)$ is the corresponding scaling exponents in amplitude frequency space, if the scale invariant is hold in the original time series. Equation (7) therefore provides an alternative way to extract the scaling exponents.

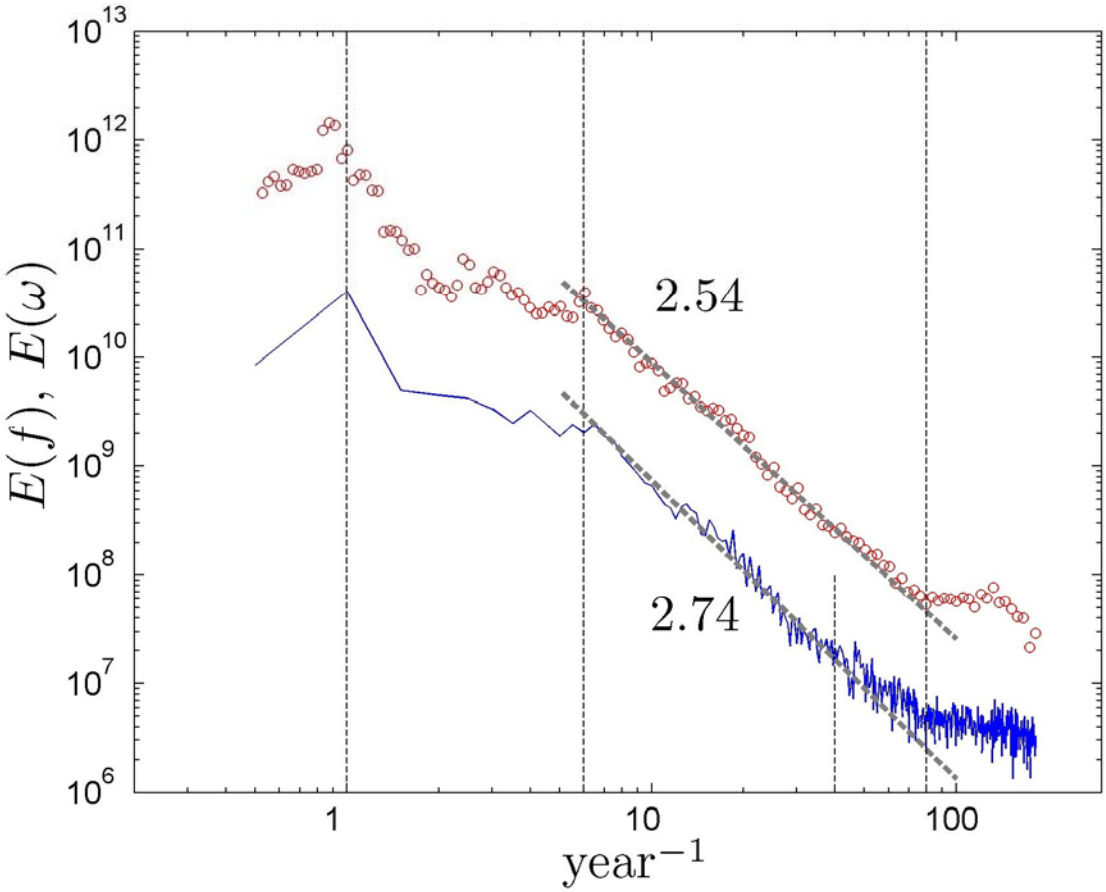

Figure 2. Comparison of the Hilbert marginal spectrum (circle) and Fourier spectrum (solid line). Power law behavior is observed on range $6 < \omega < 80$ year^{-1}, or $4.5 - 60$ days, which is marked by the vertical dash-dot lines. The scaling exponent values are 2.54 and 2.74 for Hilbert spectrum and Fourier spectrum respectively.

4 RESULTS AND DISCUSSIONS

We compare the Hilbert marginal spectrum and Fourier spectrum in Figure 2. Both of them capture the annual cycle and show power law behaviour on range $6 < \omega < 80$ year^{-1} or from $4.5 - 60$ days, with scaling exponent 2.54 and 2.74 respectively. Since we are concerned with the scaling properties in the above range, we then divided the entire time series into 16 segments, each one has 2×365 points, 2 years each. The arbitrary order Hilbert marginal spectra are shown in Figure 3, where $q = 0$, 1, 3, 4, 5 and 6. Power law behaviour is observed in all cases on range $6 < \omega < 80$ year^{-1}. The corresponding scaling exponents $\xi(q)$ are estimated on this range by using least square fitting with 95% confidence limit. We then show the scaling exponents $\xi(q)$ (circle) in Figure 4. The curve is concave, which indicates the multifractal properties of the river flow discharge. For comparison, we also show a reference line *qH* with $H = \xi(1) - 1 = 0.84$ (solid line). We also display the departure from this reference in insert, the shape is similar to turbulent ones (Frisch, 1995, Benzi et al. 1995).

EMD decomposes the original time series into different scales directly. Both EMD and HSA have very local ability. Therefore the arbitrary order Hilbert spectral analysis measures the scale invariant properties in a quite elegant way. They can constrain the nonlinear and nonstationary effects both in real space and frequency space.

We want emphasize some points on the sample size here. The total length of the data is 111828 points. It is a large database, but may not be sufficiently large to ensure the statistical convergence of higher order moments. However, we observed the power law behavior even in order six here. Of course the scaling exponents will fluctuate slightly with the sample size. But the shape/trend of the scaling exponents shall not be modified.

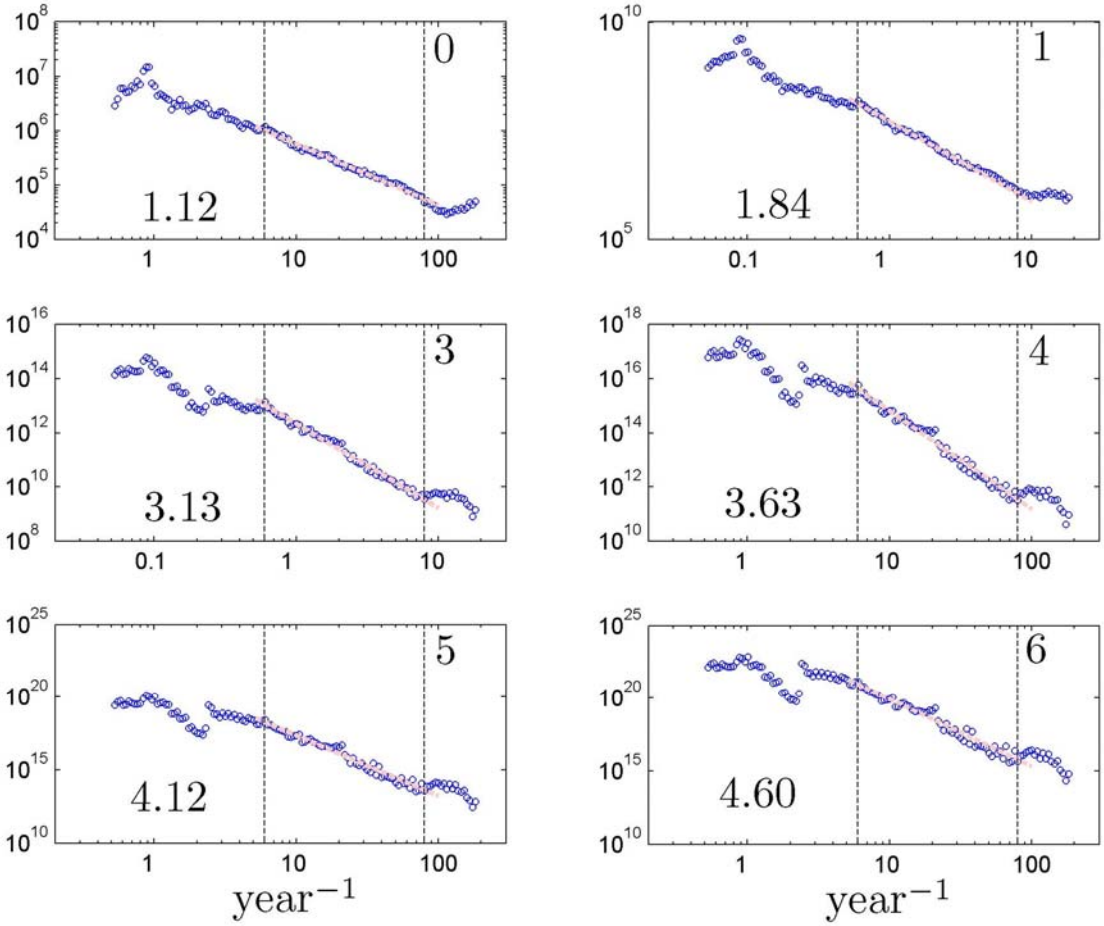

Figure 3. Arbitrary order Hilbert marginal amplitude spectra, $q = 0$, 1, 3, 4, 5 and 6. Power law is observed in all cases on range $6 < \omega < 80$ year^{-1}. The vertical dash lines indicate the power law range. The corresponding scaling value are shown in each figure.

5 CONCLUSION

In this work we have prsented the analysis of a long (32 years) time series of daily river flow data, recorded in the Seine River (France). We applied a new data analysis technique coming from turbulence studies and time series analysis, namely arbitrary order Hilbert spectral analysis. It is an extended version of the Hilbert-Huang transform we developed in order to detecting the intermittency effects to study the intermittency. This new method estimates the scale invariant directly by calculating the arbitrary order Hilbert marginal spectra, the marginal moments of the joint pdf $p(\omega, A)$. Power law behaviour is then observed in the first six order arbitrary order Hilbert marginal spectra on range $4.5 - 60$ days. The shape/trend of the scaling exponents we estimated here is similar with the turbulent ones for small scales, and confirm the multifractal nature of the river flow discharge.

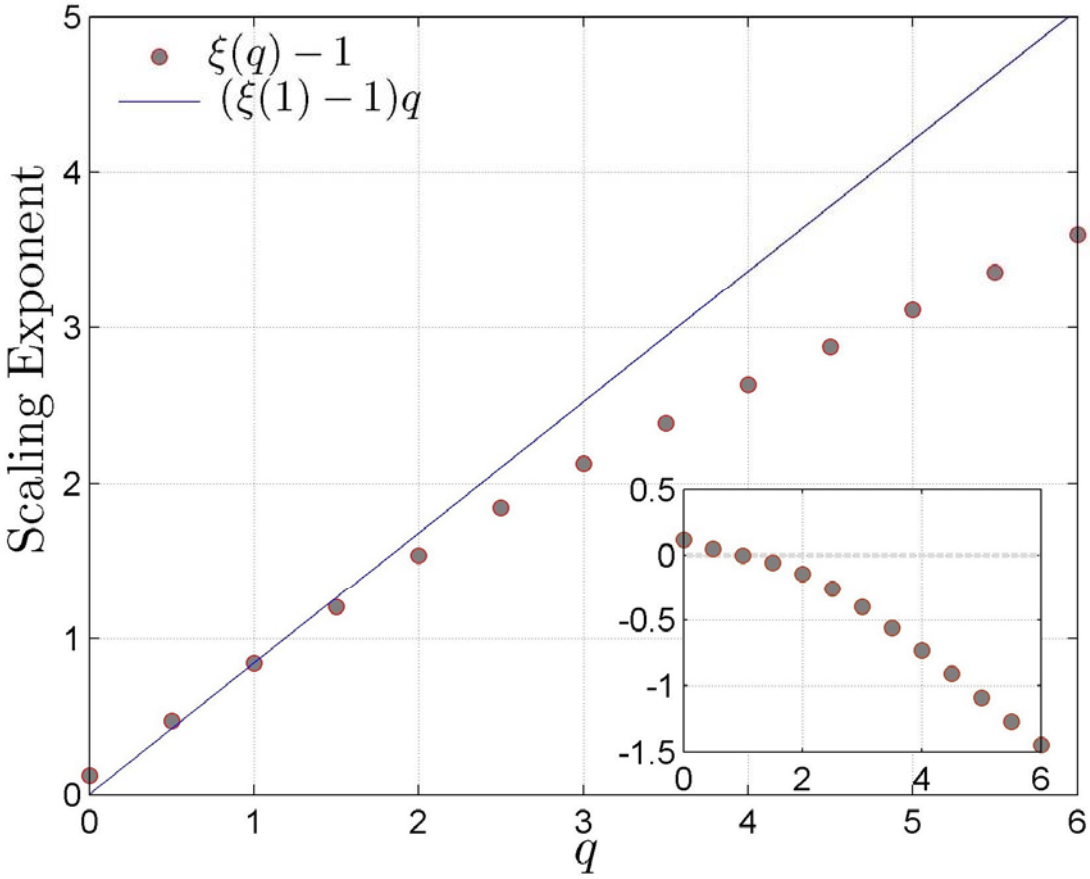

Figure 4. Scaling exponents $\xi(q)-1$. The inset shows the departure from the reference line *qH* with $H=\xi(1)-1=0.84$.

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