Hilbert-Huang Transformation and its application in optical flow evaluation

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Abstrakt

Článek se zabývá možnosti výpočtu optického toku pomocí Hilber-Huangovi transformace. Je zde také naznačen další vývoj studijního záměru pomocí této metody. Uvedeno je také srovnáni s metodou Lucas-Kanade.

Klíčová slova

Optický tok, metoda Lucas-Kanade, The empirical mode decomposition, EMD, Hilber spectral analysis, HSA, Hilbert-Huang Transform, HHT, Intrinsic mode function, IMF, Shifting process, Hilbert Spectral Analysis,

1. Introduction

Images have long held a fascination for us, for our eyes constantly input a stream of data into our minds from the world around us. We are processing images to obtain distance, size, color, orientation, along with beauty or even a sense of danger. We are always analyzing and characterizing drawings, inscriptions, paintings. We are trying to assign them a level of value and appreciation accordingly. Our minds are capable of image processing of the highest order. In recent times, it has become technically possible to obtain images that are more than just a picture. Images are actually an array of numbers of high precision that represent point-wise measurements over an area. Nowadays Images are not just the gray scale value in a scanned photograph, but a detailed measure of electromagnetic wavelength and intensity representing color. These arrays of numbers are handled easily by computer and can thus be displayed, printed, and viewed as an image, while representing a reality that our own eyes could never see. The article is going to analyze possibilities of image processing using Hilbert-Huang transformation. HHT is abbreviation of this name. HHT contains of from two steps. These steps mentioned early in the following text. This recently developed empirical mode decomposition/Hilbert-Huang transform (EMD/HHT) for the analysis of nonlinear and non-stationary data has been extended to include the analysis of image data. Because image data can be expressed in terms of an array of rows and columns, this robust concept is applied to these arrays row by row. Each slice of the data image, either row or column-wise, represents local variations of the image being analyzed. The article is trying to think about possibilities and limitations in image analysis using Hilbert-Huang Transform. In many cases Hilbert Huang transform could be a challenge for doing image processing faster. Thus, the EMD/HHT approach is especially well-suited for image data, giving frequencies, inverse distances or wave numbers as a function of time or distance, along with the amplitudes or energy values associated with these, as well as a sharp identification of embedded structures. The next part describes HHT more closely.

2. Hilbert-Huang Transform introduction

The Hilbert-Huang transform (HHT), a NASA's designated name, is proposed by Huang et al.(1996, 1998, 1999, 2003).It is the result of the empirical mode decomposition (EMD) and the Hilbert spectral analysis (HSA). The HHT uses the EMD method to decompose a signal into so-called intrinsic mode function, and uses the HSA method to obtain instantaneous frequency data. The HHT provides a new method of analyzing nonstationary and nonlinear time series data. This article focuses on HHT for image analysis and on its limitations for this purpose.

2.1. The empirical mode decomposition (EMD)

The EMD method [3][11] is a necessary step to reduce any given data into a collection of intrinsic mode functions (IMF) to which the Hilbert spectral analysis can be applied. An IMF is defined as a function that satisfies the following requirements:

1. In the whole data set, the number of extrema and the number of zero-crossings must either be equal or differ at most by one.
2. At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

Therefore, an IMF represents a simple oscillatory mode as a counterpart to the simple harmonic function, but it is much more general: instead of constant amplitude and frequency in a simple harmonic component, an IMF can have variable amplitude and frequency along the time axis.

The procedure of extracting an IMF is called sifting. The sifting process is as follows:

1. Identify all the local extrema in the test data.
2. Connect all the local maxima by a cubic spline line as the upper envelope.
3. Connect all the local maxima by a cubic spline line as the upper envelope.

The upper and lower envelopes should cover all the data between them. Their mean value is m_1. The difference between the data and m_1 is the first component h_1 (1):

$$X(t) - m_1 = h_1 \tag{1}$$

Ideally, h_1 should satisfy the definition of an IMF, for the construction of h_1 described above should have made it symmetric and having all maxima positive and all minima negative. After the first round of sifting, the crest may become a local maximum. New extrema generated in this way actually reveal the proper modes lost in the initial examination. In the subsequent shifting process, h_1 can only be treated as a proto-IMF. In the next step (2), it is treated as the data, then

$$h_1 - m_{11} = h_{11} \tag{2}$$

After repeated sifting up to k times, h_1 becomes an IMF 0, which is

$$h_{1(k-1)} - m_{1k} = h_{1k}$$

Then, it is designated as the first IMF component (3) from the data:

$$c_1 = h_{1k} \tag{3}$$

2.2. The stopping criterion of the shifting process

The stopping criterion determines the number of sifting steps to produce an IMF [3][8]. Two different stopping criterions have been used traditionally:

1. The first criterion (4)is proposed by Huang et al. (1998). It similar to the Cauchy convergence test, and we define a sum of the difference, SD, as

$$SD_k = \frac{\sum_{t=0}^{T} \left| h_{k-1}(t) - h_k(t) \right|^2}{\sum_{t=0}^{T} h_{k-1}^2(t)} \tag{4}$$

Then the shifting process stops when SD is smaller than a pre-given value.

2. The second criterion is based on the number called the S-number, which is defined as the number of consecutive siftings when the numbers of zero-crossings and extrema are equal or at most differing by one. Specifically, an S-number is pre-selected. The sifting process will stop only if for S consecutive times the numbers of zero-crossings and extrema stay the same, and are equal or at most differ by one.

Once a stoppage criterion is selected, the first IMF, c_1, can be obtained. Overall, c_1 should contain the finest scale or the shortest period component of the signal. We can, then, separate c_1 from the rest of the data by $X(t) - c_1 = r_1$. Since the residue, r_1, still contains longer period variations in the data, it is treated as the new data and subjected to the same sifting process as described above.
This procedure can be repeated to all the subsequent r_j's, and the result is (5)

$$r_{n-1} - c_n = r_n \tag{5}$$

The sifting process stops finally when the residue, r_n, becomes a monotonic function from which no more IMF can be extracted. From the above equations, we can induce that (6)

$$X(t) = \sum_{j=1}^{n} c_j + r_n \tag{6}$$

Thus, a decomposition of the data into n-empirical modes is achieved. The components of the EMD are usually physically meaningful, for the characteristic scales are defined by the physical data. Flandrin et al. (2003) and Wu and Huang (2004) have shown that the EMD is equivalent to a dyadic filter bank.

2.3. Instantaneous frequency

Instead of fact that notations like instantaneous energy or instantaneous envelope of signal are well accepted notation of instantaneous frequency has been highly controversial. There are two basic difficulties with accepting the idea of an instantaneous frequency as follows. The first is connected with deeply entrenched influence of the Fourier spectral analysis [12]. The second difficulties arise from the non-unique way in defining the notation instantaneous frequency. Nevertheless, these difficulties now exist as ‘paradoxes’ because of introduction of Hilbert transform usage [12] . For an arbitrary time series, $X(t)$, we can always have its Hilbert Transform $Y(t)$, as

$$Y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x(\tau)}{t - \tau} d\tau \tag{7}$$

P indicates the Cauchy principal value. This transform exists for all functions of class L^p [13]. With defined $X(t)$ and $Y(t)$ we can form complex conjugate pairs. By doing this we have analytical signal, $Z(t)$, as

$Z(t) = X(t) + iY(t) = a(t)e^{i\theta(t)}$ in which $a(t) = \sqrt{X(t)^2 + Y(t)^2}$ and $\theta_{(t)} = \arctan(\frac{Y(t)}{X(t)})$

Currently is for us more important conclusion where is defined instantaneous frequency as follow:

$$\varpi = \frac{d\theta_{(t)}}{dt} \tag{8}$$

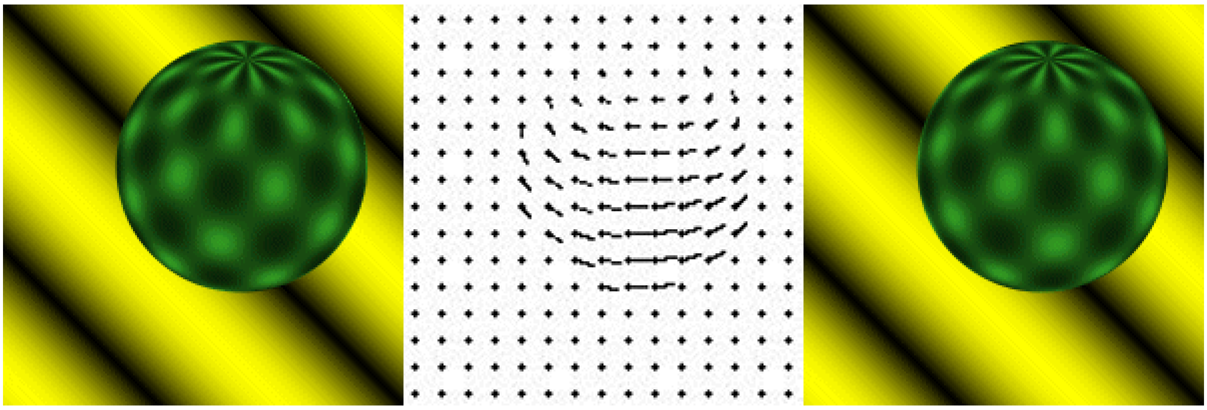
Whole definition is mention in literature [12]. This short introduction should bring out and do more understandable notation of instantaneous frequency for better understanding to the next text.

2.4. Hilbert spectral analysis

Having obtained the intrinsic mode function components, the instantaneous frequency can be computed using the Hilbert transform / Hilbert spectral analysis [3][8][9] 0. The instantaneous frequency is the term for derivate of the phase as you saw above. After performing the Hilbert transform on each IMF component, the original data can be expressed as the real part. Real part is used in the following form 0:

$$X(t) = \mathrm{Re}\,al\sum_{j=1}^{n} a_j(t)e^{i\int \omega_j(t)dt} \tag{9}$$

3. Optical flow analysis

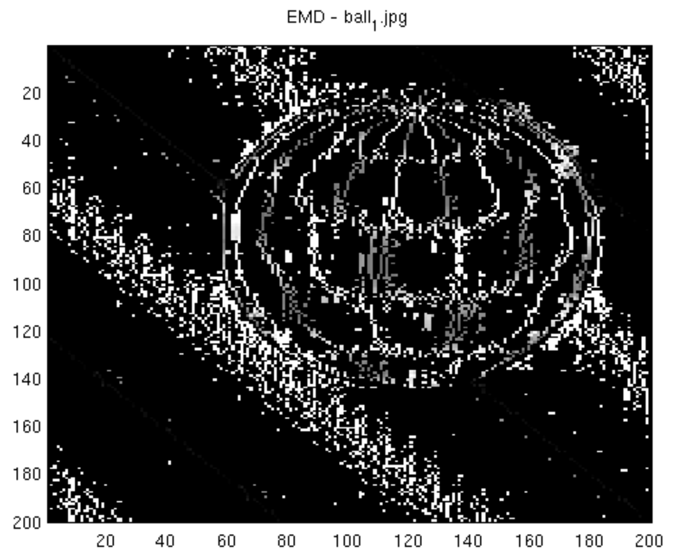


Picture 1 - Optical flow evaluation by Lucas-Kanade method

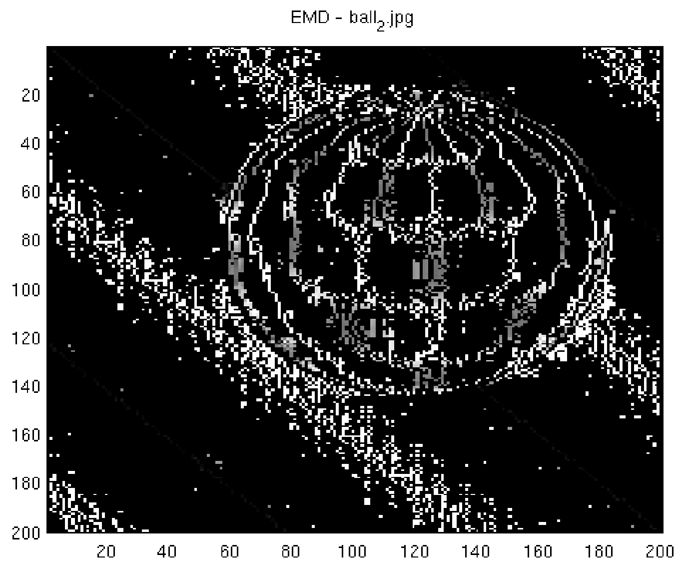
In robotics, optical flow [1][2][4] displayed on Picture 1 is a well known term. By optical flow we can analyze for example machine movement with regards to scene. In common way optical flow is evaluated from a series of consequential images. The mage interval between all images should be very short. Images of ball rotation are taken as example in the article. Ball rotation has left direction and for evaluation is used well known Lucas-Kanade method [1][2] . Lucas-Kanade method has to be realized with some limitations and simplifications according to scene. It introduces an additional term to the optical flow by assuming the flow to be constant in a local neighborhood around the central voxel[7] (This is analogous to a pixel, which represents 2D images data) under consideration at any given time. Based on this we are able to evaluate optical flow. I do recommend studying referenced literature for better understanding. The article is now going to have deeper look at optical flow evaluation using HHT method. HHT method is used only in the simplest way. There won't be used any stoppage criteria. Analysis which is going to be mentioned is using only the first iteration for getting IMF dataset h_1 (1). It means value m_1 is evaluated by the well known basic equation $m_1 = \frac{val_1 + val_2}{2}$. Hilbert transform is applied after then.

3.1. Hilbert-Huang transforms usage

For analysis are taken similar ball images as in Lucas-Kanade method decryption. Images order was only switched. Now ball rotation is to the right direction. Matlab is used as the software for whole analysis. Both images were converted to gray scale spectrum and opened inside. Images were also stored as a brightness matrix. Gray scale spectrum is used for simplification of the whole analysis. The first part EMD (empirical mode decomposition) of HHT method was applied over whole images. Data were read by rows. From both images were taken only maximums and minimums. Other parts are displaced by black color. As is seen from both images information about ball movement stay still saved. Evaluation of maximum and minimum values works as kind of filter.

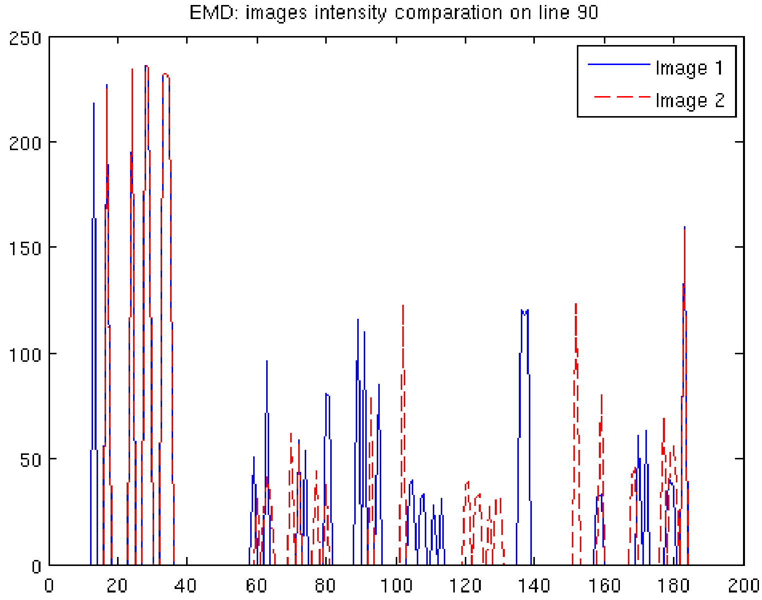


Picture 2 - Filtered Image 1: only maxima and minima displayed



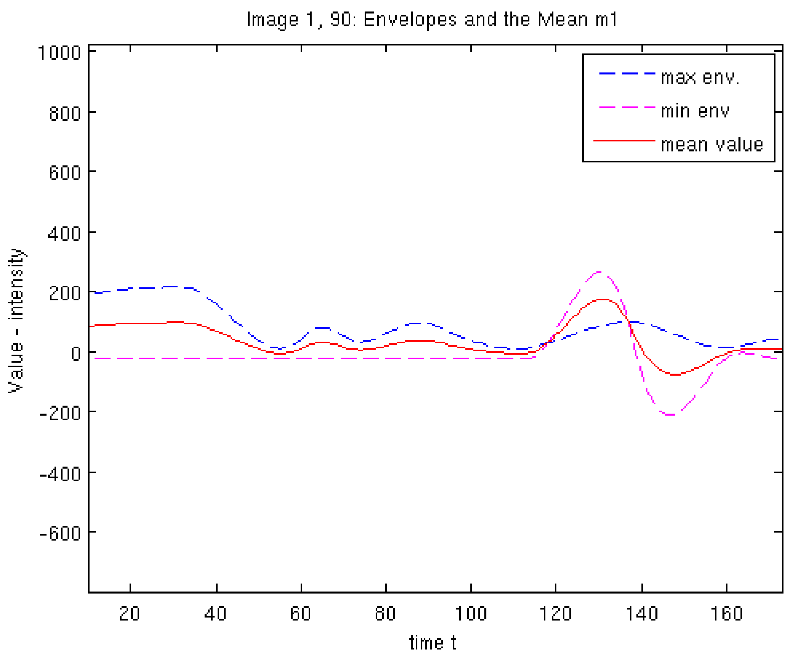
Picture 3 - Filtered Image 2: only maxima and minima are displayed

In the next steps an appropriate row for evaluation is chosen. This row has to be over the ball and should contain information about ball rotation. For these purposes is taken row number 90 for both images. Brightness of both images is containing similar parts from which could be seen potential movement to right. This estimation is done by knowledge of analyzed images.

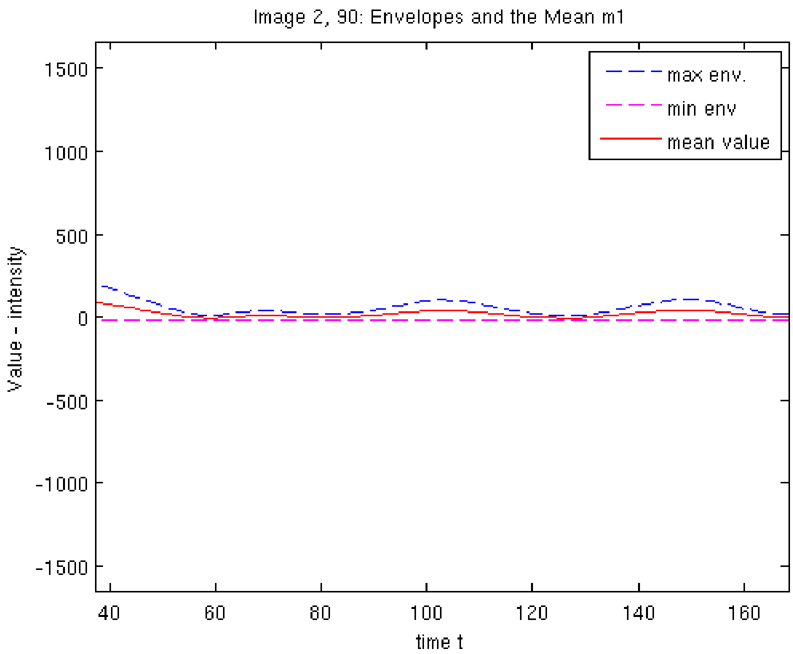


Picture 4 *- Brightness comparison on line 90 from previous images*

Next images are displaying splines creation procedure with defined mean values on both images.

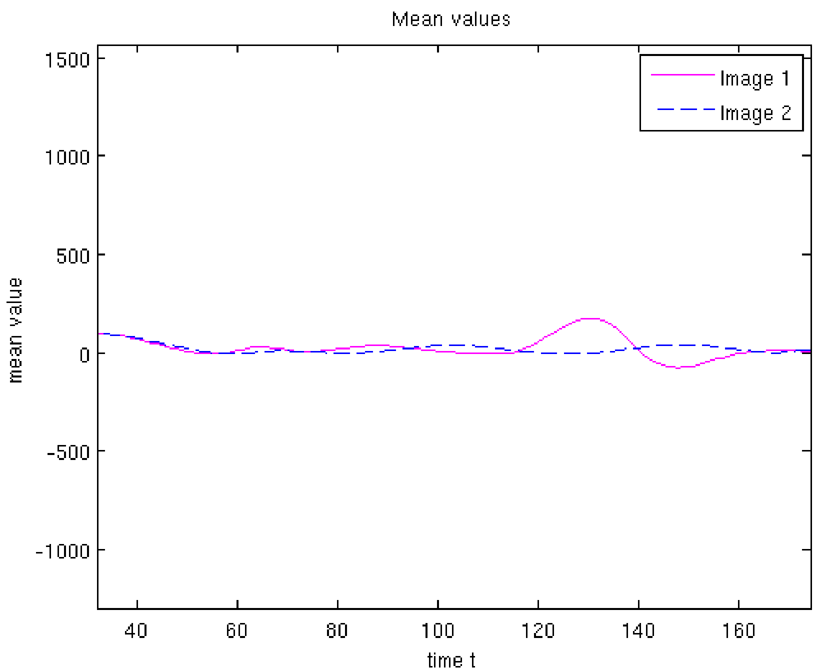


Picture 5 – *Image 1: spline building from gained maxima and minima points*



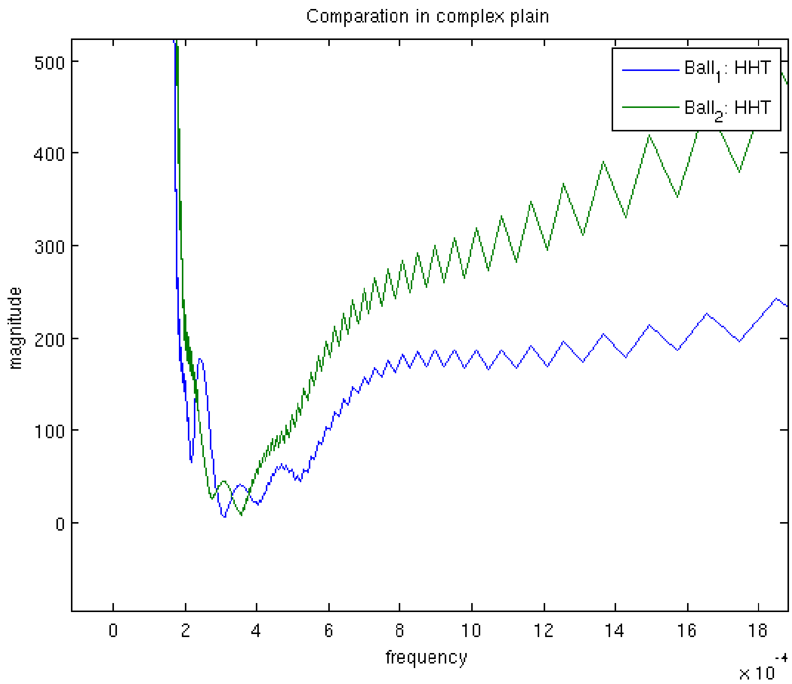
Picture 6 *- Image 2: spline building from gained maxima and minima points*

Picture 7 is displaying mean values comparison for both images. This step is done only on brightness knowledge. Displayed interval of mean value flows is the interval of brightness changes.



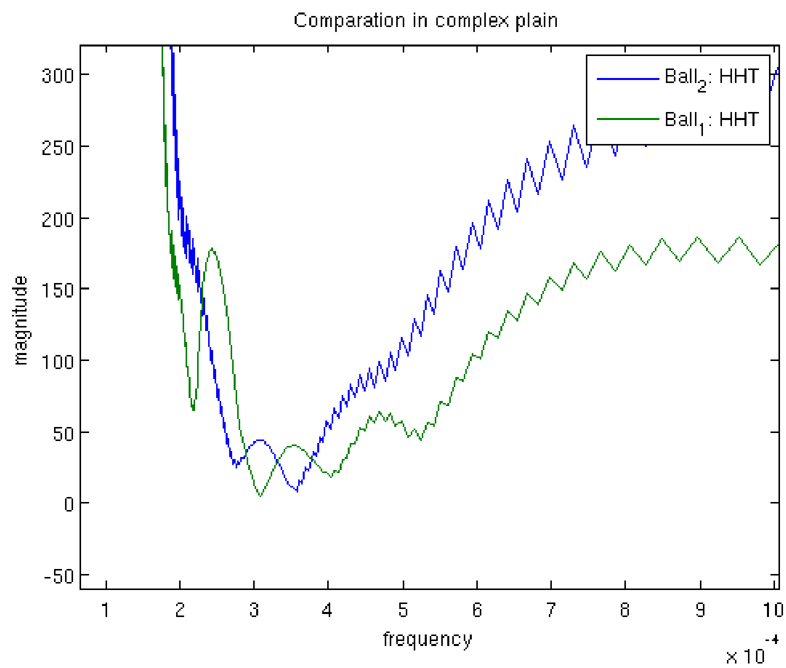
Picture 7 *- mean values from both images comparison*

Having prepared and defined all IMF (intrinsic mode functions) components we can apply Hilbert Transformation. Hilbert transform moves real numbers to complex plain and the result could be displayed as dependency of frequency [10] on magnitude. Frequency is evaluated in dependency of x position on appropriate image. In our example it is line number 90.



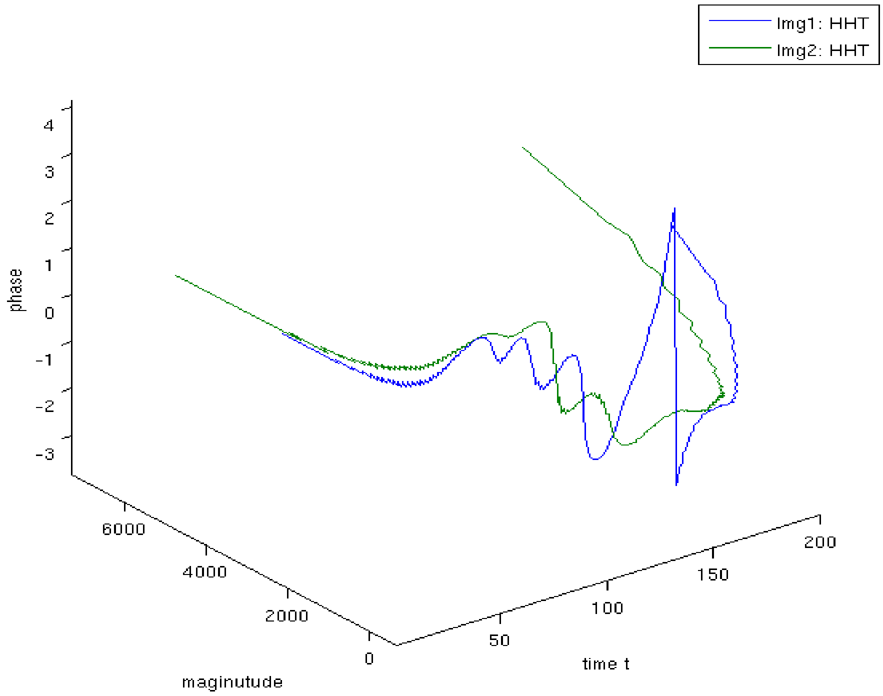
***Picture 8 –** Frequency dependency on magnitude*

Next images are displayed frequency dependency on magnitude when ball images are switched. As you seen from the Picture 8 frequency is different for both. When Images position is switched frequency is switched too as you see it on Picture 9. From these two images we have seen changes of frequency which are done. Similar tests were done over whole images for all its rows. Results were till same.



***Picture 9 -** Frequency dependency on magnitude*

This rapid change could be understood that something is happening. This deviation could be read as optical flow change but there is no proof for this kind of interpretation. From Picture 10 we have seen 3D comparison of time (x axis value), magnitude and phase. In picture are seen curves deviations. These deviations depend on numbers of points placed on balls. Divergence in curves behaviors could be read as ball position change in time in the appropriate direction. This could be understood as no similar conditions during capture procedure. It means we have fail in reaching fulfilling conditions. Brightens differences or position could led to failure.



Picture 10 – *3D plot time, magnitude and phase dependency*

4. Conclusion

This article wants to show that something happen in appropriate data analysis using Hilbert-Huang transform. From all images you have seen values movement based on the ball rotation. To define results in a more general way more tests have to be done. Based on images placed inside the article I believe it is possible to get optical flow in a much easier way than from Lucal-Kanade method.

Literature

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