

# Hilbert—Huang Transform and Its Applications

**Editors** 

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### PREFACE

Empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA) represent a desperate attempt to break the suffocating hold on data analysis by the twin assumptions of linearity and stationarity. To analyze the data from nonlinear and non-stationary processes, various attempts such as Spectrograms, Wavelet analysis, and the Wigner-Ville distribution have been made, but the EMD-HSA approach is unique and different from the existing methods of data analysis. The EMD-HAS is truly an adaptive time-frequency analysis. It does not require an a priori functional basis. Instead, the basis functions are derived adaptively from the data by the EMD sifting procedures; the instantaneous frequencies are computed from derivatives of the phase functions of the Hilbert transform of the basis functions; the final result is presented in the time-frequency space. The EMD-HSA is a magnifying glass for analyzing the data from nonlinear and non-stationary processes. The EMD-HSA results are intriguing and are no longer shackled by spurious harmonics (the artifacts of imposing a linearity property on a nonlinear system) or limited by the uncertainty principle (the consequence of Fourier transform pairs in data analysis).

EMD-HSA was originally designed in 1995 specifically to study water surface wave evolution, the phenomenon of high frequency waves with short fetch evolving into low frequency waves at long fetch. With the EMD-HSA method, it was found that the evolution of the waves was not continuous but abrupt, discrete and local. Subsequently, NEH spent two years visiting Caltech at the invitation of Professor Theodore Y. Wu. Under the guidance of Professor Wu and Professor Owen M. Phillips of the Johns Hopkins University, the EMD-HSA method was further developed and various applications explored. Professor Wu designated the method as the Hilbert-Huang Transform (HHT), a name later adopted by NASA to avoid the awkward name of EMD-HSA. It is only fair to say that the HHT would not have been developed without the encouragement and guidance of Professors Wu and Phillips.

The HHT's power and effectiveness in data analysis have been demonstrated by its successful application to many important problems covering engineering, biomedical, financial and geophysical data. The mathematical development of the HHT, however, is undergoing the same path as other significant and historical data analysis methods as in Fourier analysis and wavelet analysis: Applications are leading to development, and the mathematical theories are following, since the methods were motivated by applications. Mathematicians' apparent interest in the HHT motivated our organization of an HHT mini-symposium at the joint meeting between

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the Society for Industrial and Applied Mathematics and the Canadian Applied and Industrial Mathematics Society in June of 2003 at Montreal.

This book contains most of the presentations made at the mini-symposium with some additions. The book contents are divided into two groups: the theoretical aspects and the applications, with the applications further grouped into geophysics, structural safety, and visualization. In the theoretical aspects, the chapters cover the attempts of mathematicians to apply rigor to the empirical method such as the representation of the IMF by B-spline functions, filter based decompositions, and the statistical characteristics of the IMFs. This book also represents a plea for help from the mathematical community. A list of outstanding mathematical problems is given in Chapter 1. The chapters on applications include the correction of satellite orbit drifting, detection of failure of highway bridges and other structures, discoveries of the patterns and anomalies in climate data, and calculation of the instantaneous frequency of water waves. The objectives of the book are to provide HHT users with a collection of successful HHT applications, to supply graduate students and researchers with an HHT tutorial, and to inform data analysis mathematicians of the outstanding mathematical problems of HHT.

This book is intended as a reference for anyone who are involved in signal analysis by processing data from nonlinear and non-stationary systems. Although each chapter is independent from the others, it is sufficiently pedagogical so that every single chapter or the entire book is suitable as a part of a graduate course on signal analysis. To use this book efficiently, the readers should have background knowledge of calculus, Fourier transform, numerical analysis and differential equations. The HHT algorithm has been patented by NASA; non-commercial users may obtain it at the website: <a href="http://techtransfer.gsfc.nasa.gov">http://techtransfer.gsfc.nasa.gov</a>.

Much effort went into compiling this collection of papers into a book form. In this processes, we owe our gratitude to Dr. Dean Duffy for his skillful editing and typesetting, and without his efficient and professional work, this book would not have been possible.

> Norden E. Huang and Samuel S. P. Shen Greenbelt, Maryland

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